ISMIP HEINO

Ice Sheet Model Intercomparison Project - Heinrich Event INtercOmparison

REINHARD CALOV Potsdam Institute for Climate Impact Research, Potsdam, Germany

RALF GREVE Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan

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1 Introduction

Heinrich events (HEs) are large scale surges of the Laurentide ice sheet over Hudson Bay and Hudson Strait. They are important for the glaciological research as well as in the climatological context. Payne (1995), Greve and MacAyeal (1996) and Hindmarsh and Le Meur (2001) modeled self-sustained oscillations like HEs with 2D ice sheet models (one vertical and one horizontal direction). Marshall and Clarke (1997) first simulated such oscillations with a 3D model of the Laurentide ice sheet. Recently, Calov et al. (2002) simulated HEs, whose ice-volume variation matches well with the sea-level change of about 10 m derived from corals reported by Cappell et al. (2002). Here, we would like to test how far other 3D ice sheet models are able to simulate such large scale surges and, if they do, what are the differences between the given model results.

In detail we wish to know:

- Are there HEs in the given model, and under which circumstances do they appear?
- How long do the events last and how long is the recurrence time between events? Are the events periodical or quasi-periodical?
- How large are the changes in ice volume, ice thickness, homologous basal temperature and basal temperate area?
- How do the ice-surface elevation, homologous basal temperature, sliding and surface velocity look like at certain time slices before, during and after the events?

We think that there are three requirements to model HEs: (i) a thermomechanical model is needed, (ii) the basal sliding must be sufficiently high, and (iii) a sediment mask which serves as a geological template allowing for high sliding velocities must be prescribed. In our model (Calov et al., 2002) Heinrich events evolve through an activation and deactivation wave (Fowler and Schiavi, 1998) of the temperate basal area. This is why the temperate basal regions are considered in this comparison model also.

In some parts we follow the setup of the sliding experiments of EISMINT phase 2 (Payne et al., 2000), but there are also important differences, namely the forcing and the masks which prescribe the regions where intensive sliding may appear if the basal temperature reaches the pressure melting point. The experiments can be done with shallow-ice-approximation, higher-order and full-Stokes models.

2 Model domain

The model domain (Fig. 1) is a flat horizontal square of the size 4000×4000 km,

$$x = 0 \dots 4000 \,\mathrm{km}, \quad y = 0 \dots 4000 \,\mathrm{km},$$
 (1)

discretized by a 50-km grid. This leads to 81×81 grid points indexed by

$$i = 1 \dots 81, \quad j = 1 \dots 81.$$
 (2)

The choice of the vertical resolution as well as the time step are left open. The land area prone to glaciation is situated within the circle

$$d = \sqrt{(x - \hat{x})^2 + (y - \hat{y})^2} < R,$$
(3)

where the center of the domain has the position

$$(\hat{x}, \hat{y}) = (2000 \,\mathrm{km}, \, 2000 \,\mathrm{km}),$$
(4)

d denotes the distance from the center, and the radius is R = 2000 km. The area outside the circle is assumed to be ocean, where the ice thickness always equals zero.

Within the land area defined by (3) two different types of bedrock are distinguished: (i) hard rock, and (ii) soft sediment. Soft sediment is assumed for the regions of "Hudson Bay" within the square (including the borders)

$$A = (2300 \text{ km}, 2500 \text{ km}), \quad C = (3300 \text{ km}, 2500 \text{ km}),$$

$$B = (2300 \text{ km}, 1500 \text{ km}), \quad D = (3300 \text{ km}, 1500 \text{ km}),$$

and "Hudson Strait" within the rectangle (including the borders)

$$E = (3300 \text{ km}, 2100 \text{ km}), \qquad G = (4000 \text{ km}, 2100 \text{ km}),$$
$$F = (3300 \text{ km}, 1900 \text{ km}), \qquad H = (4000 \text{ km}, 1900 \text{ km}).$$



Figure 1: HEINO model domain. The land area is shown in white, the ocean is greyshaded. The areas inside the square ABCD ("Hudson Bay") and the rectangle EFGH ("Hudson Strait") correspond to soft sediment. The remaining land area is hard rock.

For the remaining part of the model domain, hard rock is assumed (see Fig. 1).

A supplementary file mask.dat is provided which marks hard-rock grid points with the label "1", soft-sediment grid points with "2" and ocean grid points with "0", respectively (App. A). This mask file can be read with the FORTRAN 77/90/95 subroutine read_mask.f which is supplied also.

3 Ice flow

For the flow law of ice, the usual power law ("Glen's flow law")

$$\dot{\boldsymbol{\epsilon}} = EA(T')\sigma^{n-1}\boldsymbol{\sigma} \tag{5}$$

is used, with the strain rate tensor $\dot{\boldsymbol{\epsilon}}$, the stress deviator $\boldsymbol{\sigma}$, the effective shear stress $\sigma = [\frac{1}{2} \operatorname{tr}(\boldsymbol{\sigma}^2)]^{1/2}$, the flow-enhancement factor E = 3 and the power exponent n = 3. The stress deviator reads

$$\boldsymbol{\sigma} = \boldsymbol{t} + p \boldsymbol{1}, \quad p = -\frac{1}{3} \operatorname{tr}(\boldsymbol{t}), \tag{6}$$

with the stress tensor t, the unit tensor 1 and the pressure p. Above equations assume incompressibility of ice.

The rate factor A(T') depends on the homologous temperature T' (temperature relative to pressure melting as defined in Sect. 4) via the Arrhenius relation

$$A(T') = A_0 \exp\left(-\frac{Q}{RT'}\right),\tag{7}$$

where $R = 8.314 \,\mathrm{J}\,\mathrm{mol}^{-1}\,\mathrm{K}^{-1}$ is the universal gas constant, and

$$A_{0} = 3.61 \times 10^{-13} \,\mathrm{Pa^{-3} \, s^{-1}} \qquad \text{for } T' < 263.15 \,\mathrm{K},$$

$$Q = 6.0 \times 10^{4} \,\mathrm{J \, mol^{-1}} \qquad \text{for } T' < 263.15 \,\mathrm{K},$$

$$A_{0} = 1.73 \times 10^{3} \,\mathrm{Pa^{-3} \, s^{-1}} \qquad \text{for } T' \ge 263.15 \,\mathrm{K}.$$

$$(8)$$

$$Q = 13.9 \times 10^{4} \,\mathrm{J \, mol^{-1}} \qquad \text{for } T' \ge 263.15 \,\mathrm{K}.$$

Basal sliding is assumed to follow a non-linear sliding law on hard rock and a linear sliding law on soft sediment, which relate the basal sliding velocity $v_{\rm b}$ to the basal shear stress $\tau_{\rm b}$ (note that both quantities are vectors)

$$\boldsymbol{v}_{\rm b} = \begin{cases} -\frac{C_{\rm R}}{\rho g} \frac{|\boldsymbol{\tau}_{\rm b}|^2}{N_{\rm b}^2} \boldsymbol{\tau}_{\rm b} & \text{for } T_{\rm b} = T_{\rm pmp} \text{ and hard rock,} \\ \\ -\frac{C_{\rm S}}{\rho g} \boldsymbol{\tau}_{\rm b} & \text{for } T_{\rm b} = T_{\rm pmp} \text{ and soft sediment,} \end{cases}$$
(9)
$$\boldsymbol{0} & \text{for } T_{\rm b} < T_{\rm pmp}, \end{cases}$$

where ρ is the density of ice, g the gravity acceleration, N_b the basal normal stress, T_b the basal temperature and T_{pmp} the pressure-melting point. Since in the given model setup the ice-sheet bed is flat, we have simply $\boldsymbol{\tau}_b = -(t_{xz}^b, t_{yz}^b, 0)$ and $N_b = -t_{zz}^b$, where t_{xz}^b , t_{yz}^b and t_{zz}^b denote the xz-, yz- and zz-component of the stress tensor \boldsymbol{t} at the base. The sliding parameters for hard rock and soft sediment have the values $C_{\rm R} = 10^5 \,\mathrm{a}^{-1}$ and $C_{\rm S} = 500 \,\mathrm{a}^{-1}$, respectively. (Even though the numerical value of $C_{\rm R}$ is greater than that of $C_{\rm S}$, the resulting sediment-sliding velocities are three orders of magnitude larger that the hard-rock-sliding velocities.)

If the shallow ice approximation is employed, we have

$$\boldsymbol{\tau}_{\mathrm{b}} = \rho g H \boldsymbol{\nabla}_{\mathrm{H}} h, \quad |\boldsymbol{\tau}_{\mathrm{b}}| = \rho g H |\boldsymbol{\nabla}_{\mathrm{H}} h|, \quad N_{\mathrm{b}} = \rho g H$$
(10)

with the horizontal gradient $\nabla_{\rm H} = (\partial/\partial x, \partial/\partial y)$, and the basal sliding velocity can be expressed directly through the ice thickness H and the ice-surface elevation h via

$$\boldsymbol{v}_{\rm b} = \begin{cases} -C_{\rm R} H \, |\boldsymbol{\nabla}_{\rm H} h|^2 \, \boldsymbol{\nabla}_{\rm H} h & \text{for } T_{\rm b} = T_{\rm pmp} \text{ and hard rock,} \\ -C_{\rm S} H \, \boldsymbol{\nabla}_{\rm H} h & \text{for } T_{\rm b} = T_{\rm pmp} \text{ and soft sediment,} \\ \mathbf{0} & \text{for } T_{\rm b} < T_{\rm pmp}. \end{cases}$$
(11)

4 Homologous temperature

The homologous temperature T' (in K) is given by

$$T' = T + \beta H,\tag{12}$$

with the absolute temperature T (in K), the ice thickness H (in m) and the Clausius-Clapeyron gradient β as listed in table 1.

5 Atmospheric boundary conditions

The surface accumulation b is assumed to increase linearly from the center value $b_{\min} = 0.15$ m ice equiv. a^{-1} over the radius R = 2000 km to the margin value $b_{\max} = 0.3$ m ice equiv. a^{-1} , that is,

$$b = b_{\min} + \frac{b_{\max} - b_{\min}}{R} \times d.$$
(13)

Ablation is not considered; however, it is assumed that the coastward ice-mass flux is discharged into the surrounding ocean, where the ice thickness is zero.

The surface temperature T_s is assumed to increase with the **third power** of the distance *d* from the center of the model domain towards the margin,

$$T_{\rm s} = T_{\rm min} + S_T d^3, \tag{14}$$

with the minimum temperature $T_{\rm min} = 233.15 \,\mathrm{K}$ and the horizontal gradient $S_T = 2.5 \times 10^{-9} \,\mathrm{K \, km^{-3}}$.

6 Model runs

For all model runs the ice is built up from ice-free initial conditions (zero ice thickness) over a time of t = 0...200 ka. Since our focus is the ice sheet only, the bedrock elevation remains constant (flat) throughout the simulations (no isostasy). Similarly, the geothermal heat flux applies directly at the bottom of the ice sheet (no thermal bedrock).

We recommend the usage of rather small time steps. The reason is not only for numerical stability, but also because the migration of the activation wave is a rather fast process which needs to be sufficiently resolved in time. For the 50-km grid spacing we used a common time step of 0.25 years for both the evolutions of topography and temperature.

Table 1 summarizes the prescribed standard model parameters.

Parameter	Value
Density of ice, ρ	$910\mathrm{kg}\mathrm{m}^{-3}$
Gravity acceleration, g	$9.81{ m ms^{-2}}$
Power-law exponent, n	3
Flow-enhancement factor, E	3
Sliding parameter for hard rock, $C_{\rm R}$	$10^5 \mathrm{a}^{-1}$
Sliding parameter for soft sediment, $C_{\rm S}$	$500 \mathrm{a}^{-1}$
Geothermal heat flux, q_{geo}	$4.2\times 10^{-2}{\rm Wm^{-2}}$
Melting temperature of water, T_0	$273.15\mathrm{K}$
Heat conductivity of ice, κ	$2.1{\rm Wm^{-1}K^{-1}}$
Specific heat of ice, c	$2009Jkg^{-1}K^{-1}$
Clausius-Clapeyron gradient, β	$8.7 \times 10^{-4} {\rm K} {\rm m}^{-1}$
Latent heat of ice, L	$3.35 imes 10^5 { m J kg^{-1}}$
Universal gas constant, R	$8.314\mathrm{Jmol^{-1}K^{-1}}$
Seconds per year	$31556926\mathrm{sa^{-1}}$

Table 1: Values of the physical parameters used for HEINO.

6.1 The standard run

The standard run **ST** is defined by the settings and parameters as given above.

6.2 Variation of the surface temperature

The minimum temperature T_{\min} in equation (14) is varied by ± 10 K, resulting in the settings $T_{\min} = 223.15$ K for run **T1** and $T_{\min} = 243.15$ K for run **T2**, respectively.

6.3 Variation of the surface accumulation

The surface accumulation in equation (13) is changed by a factor of 0.5 and 2, respectively. The corresponding parameter values are $b_{\min} = 0.075$ m ice equiv. a^{-1} , $b_{\max} = 0.15$ m ice equiv. a^{-1} for run **B1** and $b_{\min} = 0.3$ m ice equiv. a^{-1} , $b_{\max} = 0.6$ m ice equiv. a^{-1} for run **B2**.

6.4 Variation of the sliding parameter for sediment

The sliding parameter for sediment $C_{\rm S}$ is varied according to $C_{\rm S} = 100 \,\mathrm{a}^{-1}$ (run S1), $C_{\rm S} = 200 \,\mathrm{a}^{-1}$ (run S2) and $C_{\rm S} = 1000 \,\mathrm{a}^{-1}$ (run S3), respectively.

7 Deliverables

All output files to be produced are named in the form

 $init_run_type_quantity.dat$

where

- *init* denotes the initials of your name,
- run is the name of the simulation (ST, T1, T2, B1, B2, S1, S2 or S3),
- *type* is ts for global time series, tss for time series over the sediment region, tsp for time series at certain grid points or pf for 2D plan form output,
- quantity specifies the required physical quantity.

7.1 Time series

The time-series data shall be written every year over the whole model time t = 0...200 ka, which leads to 200 001 data lines in each file. The common output format for a single data line in FORTRAN notation is

write(file_number, 1000) time, quantity
1000 format(2e14.6)

The variables time and quantity are of type real.

7.1.1 Global time series

The global time series consist of ice volume and temperate basal area for the whole ice sheet:

- Ice volume: File name *init_run_ts_iv.dat*, time in years, volume in 10⁶ km³.
- Temperate basal area: File name *init_run_ts_tba.dat*, time in years, area in 10⁶ km².

7.1.2 Time series over the sediment region ("Hudson Bay and Hudson Strait")

Here, we are interested in the sediment region only (which corresponds to the grid points labeled by "2" in the mask file mask.dat; see App. A). For this region the average ice thickness, the average homologous basal temperature and the temperate basal area are required:

- Average ice thickness: File name *init_run_tss_ait.dat*, time in years, thickness in km.
- Average homologous basal temperature: File name *init_run_tss_ahbt.dat*, time in years, temperature in K.
- Temperate basal area: File name *init_run_tss_tba.dat*, time in years, area in 10⁶ km².

7.1.3 Time series at certain grid points

Seven grid points over the sediment region are defined as follows,

$P_1 = (3900 \mathrm{km}, 2000 \mathrm{km}),$	$P_5 = (3200 \mathrm{km}, \ 2000 \mathrm{km}),$
$P_2 = (3800 \mathrm{km}, \ 2000 \mathrm{km}),$	$P_6 = (2900 \mathrm{km}, \ 2000 \mathrm{km}),$
$P_3 = (3700 \mathrm{km}, \ 2000 \mathrm{km}),$	$P_7 = (2600 \mathrm{km}, \ 2000 \mathrm{km}),$
$P_4 = (3500 \mathrm{km}, \ 2000 \mathrm{km}),$	

and are displayed in Fig. 2.



Figure 2: Positions on the grid in the sediment region where time series output is required. See section 7.1.3 for further explanations.

On the grid points P_1 to P_7 time-series output of ice thickness, homologous basal temperature and basal frictional heating is required:

• Ice thickness: File name *init_run_tspn_it.dat*, time in years, thickness in km.

- Homologous basal temperature: File name *init_run_tspn_hbt.dat*, time in years, temperature in K.
- Basal frictional heating $R_{\rm b} = -\tau_{\rm b} \cdot v_{\rm b}$: File name *init_run_tspn_bfh.dat*, time in years, frictional heating in W m⁻².

In the above file names, $n = 1 \dots 7$ numbers the seven points (Fig. 2).

7.2 2D plan form output

This is two dimensional output on the whole domain $(81 \times 81 \text{ grid points})$ for certain points in time (specified below) as follows (FORTRAN notation):

```
write(file_number, 2000) point_in_time
2000 format(e14.6)
do i=1, 81
    do j=1, 81
        write(file_number, 3000) i, j, field(i,j)
        end do
end do
3000 format(2i3,e14.6)
```

The variables i and j are of type integer and the variable field is of type real. Times are in years. Fields are the ice-surface elevation, the homologous basal temperature, the basal sliding velocity (x- and y-component) and the surface velocity (xand y-component):

- Ice-surface elevation: File name *init_run_pfn_ise.dat*, elevation field in km.
- Homologous basal temperature: File name *init_run_pfn_hbt.dat*, temperature field in K.
- Basal sliding velocity, x-component: File name $init_run_pfn_vxb.dat$, velocity field in m a⁻¹.
- Basal sliding velocity, *y*-component: File name *init_run_pfn_vyb.dat*, velocity field in m a⁻¹.
- Surface velocity, *x*-component: File name *init_run_pfn_vxs.dat*, velocity field in m a⁻¹.
- Surface velocity, *y*-component: File name *init_run_pfn_vys.dat*, velocity field in m a⁻¹.

In these file names, n = 1...4 refers to the times t_1 , t_2 , t_3 and t_4 of the sections 7.2.1–7.2.4. In order to produce the output for these times, we recommend to redo the model runs. Otherwise excessive disk space for the 2D fields would be required.

7.2.1 Fields when average thickness is maximal

Find the point in time t_1 when the maximum of the average ice thickness over the sediment region (section 7.1.2) **during the last 50 000 model years** of the simulation is reached (see Fig. 3 for illustration).



Figure 3: Illustration how to find the times t_1 , t_2 , t_3 and t_4 . Top panel: Spatial average ice thickness over the sediment region. The red arrows indicate the points in time when the maximum (t_1) and the minimum (t_2) of that curve have been reached, respectively. Middle panel: Average homologous basal temperature over the sediment region. The red arrow shows the point in time (t_3) when the minimum of that curve has been attained. Bottom panel: Temperate basal area over the sediment region. The red arrow shows the point in time (t_4) when the maximum of that curve has been reached.

7.2.2 Fields when average thickness is minimal

Find the point in time t_2 when the minimum of the average ice thickness over the sediment region (section 7.1.2) during the last 50 000 model years of the simulation is reached (see Fig. 3 for illustration).

7.2.3 Fields when average homologous basal temperature is minimal

Find the point in time t_3 when the minimum of the average homologous basal temperature over the sediment region (see section 7.1.2) during the last 50 000 model years of the simulation is reached (see Fig. 3 for illustration).

7.2.4 Fields when the activation wave area is maximal

Find the point in time t_4 when the maximum of temperate basal area over the sediment region (see section 7.1.2) during the last 50 000 model years of the simulation is reached (see Fig. 3 for illustration).

8 Results with SICOPOLIS

Here, our results of run **ST** computed with the model SICOPOLIS are shown to illustrate the required output. The periodicity of the ice surface elevation at the sediment grid points P_1 - P_7 increases with the distance from the ice margin (Fig. 4). It amounts to about 3 ka at point P_1 and approximately 5.5 ka at point P_7 .

The ice-surface elevation reaches about 4.5 km near the centre of the domain (Fig. 5, left side panel). It is not fully axi-symmetric, because the HEs appears before the ice sheet totally recovers. At the end of a HE the ice-surface elevation is approximately 1 km lower than before a Heinrich event (Fig. 5, right side panel). The axi-symmetry is totally broken now and the ice-sheet surface has a shape similar to a sickle.

The homologous basal temperature shows no axi-symmetry at both times: the time when the average homologous basal temperature over sediment is minimal (Fig. 6, left side panel) as well as at the time when the temperate basal area over sediment is maximal (Fig. 6, right side panel). The change from cold to warm regime over sediment can be clearly seen by comparing the two panels.

Similarly, the surface velocity over sediment is about 10 m/a in the slow cold regime and up to 3 km/a in the fast warm one (Fig. 7, left and right side panel).



Figure 4: Time series of the ice-surface elevation at certain grid points over the sediment region (Fig. 2) in run **ST**.



Figure 5: Ice-surface elevation in run **ST** at times when the average ice thickness over the sediment is maximal (time t_1 , left side panel) and minimal (time t_2 , right side panel).



Figure 6: Homologous basal temperature in run **ST** at times when the average homologous basal temperature over the sediment is minimal (time t_3 , left side panel) and when the temperate basal area over the sediment region is maximal (time t_4 , right side panel).



Figure 7: Absolute value of the surface velocity in run **ST**. Left side panel and right side panels correspond to the same times as in Fig. 6, respectively.

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A File mask.dat

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