

## Potential Evaporation and Soil Moisture in General Circulation Models

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### ABSTRACT

The parameterization of continental evaporation in many atmospheric general circulation models (GCMs) used for simulation of climate is demonstrably inconsistent with the empirical work upon which the parameterization is based. In the turbulent transfer relation for potential evaporation, the climate models employ the modeled actual temperature to evaluate the saturated surface humidity, whereas the consistent temperature is the one reflecting cooling by the hypothetical potential evaporation. A simple theoretical analysis and some direct computations, all ignoring atmospheric feedbacks, indicate that whenever the soil moisture is limited, GCM-based climate models produce rates of potential evaporation that exceed, by a factor of two or more, the rates that would be yielded by use of the consistent temperature. Further approximate analyses and supporting numerical simulations indicate that the expected value of dry-season soil moisture has a short memory relative to the annual cycle and that dry-season evaporation is therefore nearly equal to dry-season precipitation. When potential evaporation is overestimated, it follows that the soil moisture is artificially reduced by a similar factor, and actual evaporation may or may not be overestimated, depending on other details of the hydrologic parameterization. These arguments, advanced on theoretical grounds, explain the substantial, systematic differences between GCM-generated and observation-based estimates of potential evaporation rates and call into question the direct use of currently available GCM-generated values of potential evaporation in the assessment of the effects of climatic change on continental hydrology and water resources. They also provide a partial explanation of the excessively low values of summer soil moisture in GCMs and raise questions concerning the results of studies of soil-moisture changes induced by an increase of greenhouse gases. Nevertheless, an approximate analytical result suggests that the basic dependence of changes in soil moisture on changes in the atmospheric state was qualitatively preserved in those studies.

### 1. Introduction

This paper describes, evaluates, and corrects a basic conceptual inconsistency between the parameterizations of evaporation from the continents in several climate models and the empirical evidence upon which those parameterizations are based. The climate models concerned are those based on atmospheric general circulation models (GCMs) that describe evaporation from land as the product of a potential evaporation rate and a moisture availability function. These include nearly all models with a prognostic equation for soil moisture, excluding those that have recently introduced explicit vegetation into the parameterization of evaporation (Dickinson 1987; Sato et al. 1989). When there is a restriction on moisture availability in the affected models, the method of calculation of the potential evaporation rate yields a value that grossly exceeds the value consistent with the moisture availability function used, leading to artificially accelerated drying of the soil and, potentially, to distortion of the modeled sur-

face water and energy balances. This paper quantifies the differences in potential evaporation, presents the appropriate correction, and offers some speculation regarding the influence of the noted discrepancy upon the results and conclusions of investigators who have employed the concept of potential evaporation in climate models.

A problem with the definition of potential evaporation has been noted by others in recent years. Sud and Fennessy (1982) clearly distinguished between the definition used in GCMs and the definition used in the analysis of field data and pointed out that the moisture availability function used to obtain actual evaporation in the GCM should therefore differ markedly from the one derived from field data. They used a 47-day GCM simulation to infer the former from the latter and demonstrated that the global evaporation field in two 5-day integrations, apparently starting from the same initial conditions, was highly sensitive to the choice of the moisture availability function used. Brutsaert (1986) and Sellers (1987) also observed that the typical definition of potential evaporation implicit in GCMs differed from that of Penman (1948). Sellers (1987) stated that, as a result, the typical rates of potential evaporation in GCMs would be too high in arid

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regions and concluded that excessive evaporation would result. Pan (1988) attributed excessive moistening of the daytime boundary layer in the Medium Range Forecast Model of the National Meteorological Center to excessive rates of potential evaporation and introduced a Penman-like rate instead. None of these investigators assessed the magnitude of the differences in potential evaporation rates. Sud and Fennessy (1982) evaluated the effect on simulated evaporation in their 5-day initial-value problem, but the effect on simulated climatological evaporation and soil moisture has not been assessed.

## 2. Theoretical and empirical background

Manabe (1969) introduced a simple interactive model of continental water and energy balances into his general circulation model of the atmosphere. This scheme employed a single storage variable  $w$ , the soil moisture, to represent the total volume of water per unit land area stored in the top meter of soil beneath each stack of atmospheric grid points. For each time step of computation, the soil moisture was updated based on an accounting of precipitation, snowmelt, evaporation, and runoff. The evaluation of the evaporation rate from continental surfaces was based on a concept of potential evaporation. For sufficiently high soil moisture (75% or more of field capacity), evaporation was prescribed at the potential rate. For drier soil, the evaporation rate was a fraction of the potential rate, the ratio varying in direct proportion to the soil moisture. Until recently, Manabe's scheme was employed in most general circulation models capable of carrying soil moisture as a prognostic variable, including those of Hansen et al. (1983), Washington and Meehl (1984), Mitchell et al. (1987), Schlesinger and Gates (1980), and Arakawa (1972).

The basis for Manabe's scheme was the procedure used by Budyko (1956) for hydrologic computations required in the process of estimation of the energy balance of the earth's surface. Budyko computed "evaporability" or "possible evaporation" (i.e., potential evaporation expressed as mass per unit area per unit time) by means of the relation

$$E_p(T_w) = \frac{\rho}{r_a} [q_s(T_w) - q_a], \quad (1)$$

in which  $E_p$  is the potential evaporation rate,  $\rho$  is the density of air,  $r_a$  is the aerodynamic resistance of the boundary layer between the surface and the measurement level,  $q_s(T_w)$  is the specific humidity of saturated air at temperature  $T_w$  of the (hypothetically wet) evaporating surface, and  $q_a$  is specific humidity of the air at a standard level of measurement. (In Budyko's notation,  $r_a^{-1}$  was represented by a quantity  $D$  that he called the "coefficient of external diffusion.") The temperature of the evaporating surface was found by solving an energy balance equation,

$$R - G = LE_p + H, \quad (2)$$

in which  $R$  is net radiation (absorbed solar and atmospheric irradiance minus emitted surface irradiance),  $G$  is the rate of heat storage beneath the active surface,  $L$  is the latent heat of vaporization of water, and  $H$  is sensible heat flux to the atmosphere from the surface. Expanding  $R$  linearly in surface temperature and representing  $H$  by a turbulent diffusion equation similar to (1), Budyko (1956) arrived at

$$R_0 - 4\epsilon\sigma T_a^3(T_w - T_a) - G = \frac{L\rho}{r_a} [q_s(T_w) - q_a] + \frac{\rho c_p}{r_a} (T_w - T_a), \quad (3)$$

in which  $R_0$  is the difference between the actual incoming (combined shortwave and longwave) radiation and the longwave radiation that would be emitted if the surface were at the air temperature  $T_a$ ,  $\epsilon$  is the emissivity of the surface,  $\sigma$  is the Stefan-Boltzmann constant, and  $c_p$  is the specific heat of air at constant pressure.

Evaluation of (1) using the temperature yielded by (3) is analogous to the application of Penman's (1948) method for calculating evaporation from a water surface. Indeed, if the radiation terms in (3) were lumped together and presumed known, and if  $q_s(T_w)$  were linearized around the air temperature, then (1) and (3) would lead to Penman's equation. The rate yielded by (1) in connection with (3) may be considered an "apparent potential evaporation" rate (Brutsaert 1982, pp. 226-227) since it specifies a rate of evaporation that would occur if the surface were well supplied with water, but the atmospheric conditions at the measurement height were nevertheless held constant and not allowed to adjust to this rate. In actuality, if the surface were freely supplying water to the atmosphere, the air would become cooler, moister, and less turbulent, thereby modifying the values of  $T_a$ ,  $q_a$ ,  $r_a$ , and  $R_0$ .

Having determined his potential evaporation rate, Budyko (1956) employed the relation

$$E = \beta_w E_p(T_w) \quad (4)$$

to estimate the actual evaporation rate  $E$ . The moisture availability function  $\beta_w$  was determined empirically to follow the relation

$$\beta_w = \min \left[ \frac{w}{w_k}, 1 \right]. \quad (5)$$

The parameter  $w_k$  represents the value of soil moisture  $w$  at which evaporation switches from a rate controlled primarily by atmospheric conditions (the potential rate) to one limited by soil moisture. Budyko cited Alpatov (1954) in support of equating  $E$  with some sort of potential evapotranspiration rate when  $w$  exceeds 70%-80% of the field capacity of the soil, although it appears that Alpatov did not establish the equivalence

of this potential rate with the  $E_p(T_w)$  defined by Budyko. Later, however, the correspondence between Budyko's  $E_p(T_w)$  and the actual evaporation from well-watered plots of vegetation (wheat, beets, rice, and cotton) was confirmed experimentally by Savina (1957). A similar result of Tanner and Pelton (1960) established a reasonable correspondence between Penman's (1948) method applied to rough crop surfaces and actual evapotranspiration under conditions of adequate soil moisture. In support of the direct relation between  $E$  and  $w$  under soil control, Budyko (1956) cited experimental results from the Soviet Union. Suggested values of  $w_k$  for various seasons and geographic zones were given later by Budyko (1974, p. 97), with attribution to Zubenok (1968), although the latter neither reports the figures nor describes how they were determined. Zubenok (1978) reported the same values and stated that they were derived from data on soil moisture balance. There is also ample evidence in subsequent literature (e.g., Saxton and McGuinness 1982) that (4) and (5) are a reasonable empirical model for evaporation from short vegetation, though they probably overstate non-water-stressed evapotranspiration from tall vegetation due to the neglect of significant stomatal resistance to transpiration (Sceicz et al. 1969).

The implementation of Budyko's (1956) scheme commonly found in atmospheric general circulation models has a form that is similar to (4):

$$E = \beta_s E_p(T_s), \quad (6)$$

but uses a definition of potential evaporation that differs fundamentally from that of Budyko, that is,

$$E_p(T_s) = \frac{\rho}{r_a} [q_s(T_s) - q_a], \quad (7)$$

in which  $T_s$  is the actual computed surface temperature, determined from an energy balance that is similar to (3) but allows only the actual amount of evaporative cooling:

$$R_0 - 4\epsilon\sigma T_a^3(T_s - T_a) - G \\ = \beta_s \frac{L\rho}{r_a} [q_s(T_s) - q_a] + \frac{\rho C_p}{r_a} (T_s - T_a). \quad (8)$$

Here a new subscript is introduced for  $\beta$  in (6) since the different definitions of potential evaporation appearing in (4) and (6) can lead to the same rate of evaporation only if the moisture availability functions differ.

It is emphasized here that neither (4) nor (6) has been shown to be fundamentally superior to the other. Both yield the same maximum rate of evaporation, and both are capable, through a reasonable choice of moisture availability function, of representing the decreasing ability of the land to yield this maximum rate as the soil dries. Either one could, in principle, form the basis for the development of an empirical descrip-

tion of evaporation. Some quantitative aspects of the difference between the two formulations will be explored in the next section, but some qualitative observations may be made immediately. Under conditions when the actual rate of evaporation equals the potential rate, (3) and (8) have the same solution for temperature, so  $E_p(T_w)$  is the same as  $E_p(T_s)$  and both  $\beta_w$  and  $\beta_s$  are unity. Under a restricted water supply ( $w < w_k$ ), the temperature  $T_s$  exceeds the temperature  $T_w$ , and hence the rate  $E_p(T_s)$  exceeds the rate  $E_p(T_w)$ . It follows that if  $\beta_w(w)$  varies linearly from 0 to 1 as  $w$  goes from 0 to  $w_k$ , as specified by (5), then  $\beta_s(w)$  must pass through these same endpoints but must be nonlinear and lie below  $\beta_w(w)$ . Furthermore, it can be seen that the distance between the two curves will depend on such factors as absorbed radiation and aerodynamic resistance, since these factors help determine how much  $T_s$  departs from  $T_w$ .

In principle, either (4) or (6) can be the basis for modeling evaporation in a climate model. Equation (6) is somewhat easier to implement, since it does not require the solution of an additional, hypothetical energy balance equation. On the other hand, there is a substantial body of literature devoted toward the estimation of  $\beta_w$  from field measurements, and these field data have simply not been analyzed in the framework of (6), so corresponding estimates of the  $\beta_s$  function are unavailable. (Note, however, that an approximate relation between  $\beta_w$  and  $\beta_s$  is developed in the next section.)

In practice, (6) has been applied, but no special attention has been given to the estimation of  $\beta_s(w)$ . The best available information on  $\beta_w(w)$  has generally been used instead, leading to an inconsistent mixture of (4) and (6) that could be expected to overestimate actual evaporation. The rather extensive summary of  $w_k$  values given by Budyko (1974) and Zubenok (1978) is consistent, on average, with the value adopted originally by Manabe (1969), which was three-fourths of field capacity. This means, in essence, that Manabe (1969) approximated  $\beta_s(w)$  by  $\beta_w(w)$ . In most cases where subsequent modelers have departed from the  $\beta(w)$  specified by Manabe (1969), they have used curves that actually lie *above* that of Manabe (1969), further aggravating the tendency toward excessive evaporation at a given level of soil moisture. It is difficult to make a direct comparison, since different values of field capacity have also been used, but Arakawa (1972) took  $w_k$  to be one-half of field capacity, effectively increasing  $\beta$  values in his model for given levels of relative saturation of the soil. According to Carson (1982),  $w_k$  was also taken as one-half, or even one-third, of field capacity in certain GCMs of the Atmospheric Environment Service of Canada and the Meteorological Office of the United Kingdom.

Since empirical estimates of  $\beta_w$  are applicable mainly to small spatial scales, such as those associated with experimental plots in agricultural fields and in forests,

it is reasonable to ask how well (4) might apply at GCM grid scale and how spatial averaging of subgrid variability of soil moisture might modify the shape of the  $\beta_w$  function. Assume that (4) and (5) apply at each point within the area associated with a single GCM grid point, with  $w$  representing the local soil moisture and  $E$  the local evaporation rate. Ignoring subgrid variability of absorbed radiation and surface properties other than soil moisture, one can see that  $E_p(T_w)$  is constant over the region. Thus,

$$\langle E \rangle = \left\langle \min \left( \frac{w}{w_k}, 1 \right) \right\rangle E_p(T_w), \quad (9)$$

in which the angle brackets indicate a spatial average over the grid cell. If the variance of  $w$  within the grid area is very small, then this reduces to

$$\langle E \rangle = \min \left( \frac{\langle w \rangle}{w_k}, 1 \right) E_p(T_w), \quad (10)$$

and then the grid-scale  $\beta_w$  function is identical to the local function. In the limit of maximum subgrid variance of soil moisture, all points are either dry or at the maximum value of soil moisture, termed the field capacity, denoted by  $w_0$ . In this case, a fraction of the area  $\langle w \rangle / w_0$  is yielding water at the potential rate and the other fraction is yielding none. Then

$$\langle E \rangle = \left( \frac{\langle w \rangle}{w_0} \right) E_p(T_w). \quad (11)$$

Since  $w_k$  is less than  $w_0$ , the grid-scale  $\beta_w(\langle w \rangle)$  function implied by (11) lies below that implied by (10). The actual grid-scale relation should lie between the limiting cases (10) and (11), and one can therefore conclude that the grid-scale  $\beta_w$  function lies near or below the local  $\beta_w$  function. This analysis suggests that, if anything, the consideration of subgrid variability further strengthens the argument put forward above that the simultaneous use of  $E_p(T_s)$  and an empirically determined plot-scale  $\beta_w$  is inconsistent and will have a tendency to overestimate evaporation for a given level of grid-scale soil moisture.

It has been suggested above that either (4) or (6) could be used to model evaporation, provided that a consistent moisture availability function is employed. The practical argument favoring (4) is that the form of  $\beta_w$  is better known than that of  $\beta_s$ . However, in the next section an approximate relation between the two functions is developed. That result could conceivably be used to provide an estimate of  $\beta_s$  for use by modelers who wish to retain (6).

In the Appendix, the relative merits of (4) and (6), with  $\beta$  depending only on soil moisture, are discussed from a physical perspective. Based on the simple analyses presented there, it appears that there is no physical basis for choosing one over the other, except when it

is assumed that there is strong subgrid variability of soil moisture, in which case (4) is favored.

### 3. Linear analysis

It will be helpful in further developments if some solutions for  $E_p(T_w)$  and  $E_p(T_s)$  are presented here. An approximate solution of (3) may be found by introducing a linearization for  $q_s(T)$ ,

$$q_s(T) \approx q_s(T_a) + q'_s(T_a)(T - T_a), \quad (12)$$

in which  $q'_s(T_a)$  is the derivative of  $q_s$  with respect to  $T$ , evaluated at the air temperature. The substitution of (12), with  $T$  equal to  $T_w$ , into (3) yields a solution

$$T_w = T_a + \frac{R_0 - G - \frac{L\rho}{r_a} [q_s(T_a) - q_a]}{4\epsilon\sigma T_a^3 + \frac{\rho c_p}{r_a} + \frac{L\rho}{r_a} q'_s(T_a)}, \quad (13)$$

which, upon substitution into (1), using (12), yields a potential evaporation rate

$$E_p(T_w) = \{q'_s(T_a)(R_0 - G) + [4\epsilon\sigma T_a^3 + (\rho c_p / r_a)][q_s(T_a) - q_a]\} / [Lq'_s(T_a) + c_p + (4\epsilon\sigma T_a^3 r_a) / \rho]. \quad (14)$$

A similar solution procedure leads to the results

$$T_s = T_a + \frac{R_0 - G - \beta_s \frac{L\rho}{r_a} [q_s(T_a) - q_a]}{4\epsilon\sigma T_a^3 + \frac{\rho c_p}{r_a} + \beta_s \frac{L\rho}{r_a} q'_s(T_a)} \quad (15)$$

and

$$E_p(T_s) = \{q'_s(T_a)(R_0 - G) + [4\epsilon\sigma T_a^3 + (\rho c_p / r_a)][q_s(T_a) - q_a]\} / [\beta_s Lq'_s(T_a) + c_p + (4\epsilon\sigma T_a^3 r_a) / \rho]. \quad (16)$$

How much does  $E_p(T_s)$ , employed in many GCMs, differ from  $E_p(T_w)$ ? For the approximate solutions given above, the relative difference between  $E_p(T_s)$  and  $E_p(T_w)$  is

$$\frac{E_p(T_s) - E_p(T_w)}{E_p(T_w)} = \frac{L \frac{\rho}{r_a} q'_s(T_a) [1 - \beta_s]}{4\epsilon\sigma T_a^3 + \frac{\rho c_p}{r_a} + \frac{L\rho}{r_a} q'_s(T_a) \beta_s}. \quad (17)$$

In the GCMs,  $\beta_s$  is given the form of (5), so the solutions coincide for  $w$  greater than  $w_k$ . The largest relative difference between  $E_p(T_s)$  and  $E_p(T_w)$  occurs when soil moisture is negligible, and hence the actual latent heat flux is negligible. Let  $T_d$  denote the temperature of a surface experiencing no evaporative cool-

ing, and let  $E_p(T_d)$  denote the value of  $E_p(T_s)$  under these conditions. Then the maximum relative difference, denoted by  $\zeta$ , is given by setting  $\beta_s$  to zero in (17):

$$\zeta \equiv \frac{E_p(T_d) - E_p(T_w)}{E_p(T_w)} = \frac{\frac{L\rho}{r_a} q'_s(T_a)}{4\epsilon\sigma T_a^3 + \frac{\rho c_p}{r_a}} \quad (18)$$

This expression may be interpreted as a ratio of cooling rates, the numerator representing evaporative cooling of a hypothetically wet surface and the denominator representing combined cooling by the other available processes—namely, radiation and sensible heat transport. Hence, whenever evaporation would make a significant contribution to the energy balance of a wet surface under the prevailing conditions, the relative difference between the two calculated values of potential evaporation will be significant. This relative difference  $\zeta$  is plotted in Fig. 1 for a surface pressure of 101 325 Pa, an emissivity of unity, and various values of  $r_a$  and  $T_a$ . The magnitude of the difference between the two definitions of potential evaporation is obviously large, under all possible conditions, for this case where the actual latent heat flux is small compared to the available energy. It is interesting to note that, for sufficiently small aerodynamic resistance, the difference approaches an upper bound, which can be shown to be equal to the quantity  $\Delta/\gamma$ , where  $\Delta$  is the slope of the saturation vapor-pressure curve evaluated at the air temperature,

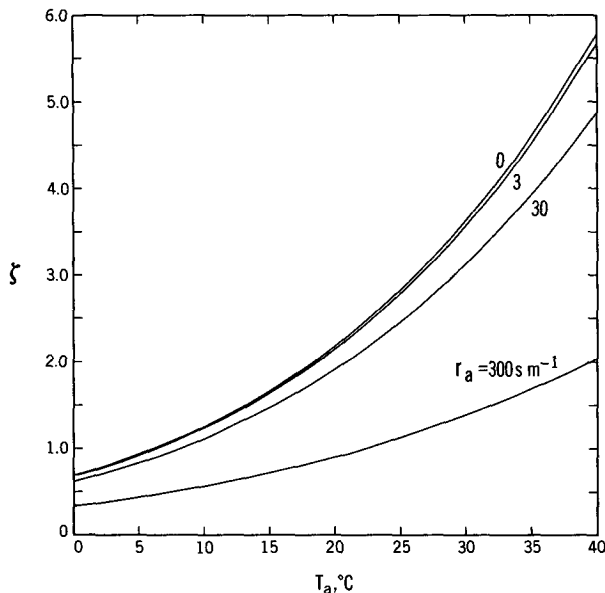


FIG. 1. Relative difference  $\zeta$  between  $E_p(T_d)$  and  $E_p(T_w)$  for various values of the aerodynamic resistance of the surface boundary layer.

$$\Delta = \frac{de_s}{dT}, \quad (19)$$

with the saturation vapor pressure  $e_s$  related to the specific humidity of saturated air and the total atmospheric pressure  $p$  by

$$q_s = 0.622 e_s/p \quad (20)$$

and  $\gamma$  is the psychrometric constant

$$\gamma = (c_p p)/(0.622 L). \quad (21)$$

Equations (17) and (18) can be combined to yield an expression for the ratio between the potential evaporation rates given by the two definitions,

$$\frac{E_p(T_s)}{E_p(T_w)} = \frac{1 + \zeta}{1 + \zeta\beta_s} \quad (22)$$

This relation will be used in later analysis. Its combination with (4) and (6) yields an approximate relation for the estimation of  $\beta_s$  from  $\beta_w$ ,

$$\beta_s = \frac{\beta_w}{1 + (1 - \beta_w)\zeta} \quad (23)$$

This implies that, if  $\beta_w$  is a function of soil moisture only, then  $\beta_s$  must depend on soil moisture and on all the factors that determine the magnitude of  $\zeta$ .

The approximate analysis presented above relies on the linear approximation (12), though it could be generalized to higher accuracy using the approach of Milly (1991), who showed that solutions of the surface energy balance equation using (12) consistently underestimate evaporation and that the amount of the bias is proportional to the surface-air temperature difference. In the current application, this means that both  $E_p(T_s)$  and  $E_p(T_w)$  are underestimated—the former more than the latter. Hence, the relative difference given by (17) is an underestimate of the true difference.

#### 4. Quantitative evaluation of potential evaporation

How much would calculated average values of the potential evaporation rate differ from those currently computed in GCMs if Budyko's (1956) definition of potential evaporation were employed? This question has been answered, ignoring atmospheric feedbacks, by direct computation using 10 years of daily output generated by a climate model of the Geophysical Fluid Dynamics Laboratory (GFDL) (T. L. Delworth, personal communication). The model is essentially that described by Manabe and Hahn (1981) and employs the standard hydrology described by Manabe (1969), which includes (6), (7), and (8), together with the assumption that  $\beta_s$  may be replaced by  $\beta_w$  as given by (5). Atmospheric motions are resolved horizontally in the spectral domain with a rhomboidal-15 wavenumber truncation, vertically by finite difference with nine levels, and temporally by a 30-minute time step. Conti-

mental hydrology is computed on a gridpoint basis with the same 30-minute time step. Solar forcing has a seasonal cycle, but no diurnal cycle. Sea surface temperatures are prescribed according to climatology, and clouds are predicted. The temperatures  $T_w$  and  $T_s$  are determined, after the simulation, by solution of (3) and (8), respectively, using once-daily model output of the other variables in those equations. Temperatures derived in this way were substituted into (1) and (7) in order to determine the corresponding rates of potential evaporation, and these rates were subsequently averaged in various ways.

Figure 2 shows annual-average, global fields of potential evaporation rates computed by the two definitions, and Fig. 3 shows zonal averages, over land and ocean separately, of the computed, annual-average potential evaporation rates. As expected from the foregoing analysis, the largest differences in computed potential evaporation occur in low-latitude deserts, where actual evaporation is low and air temperatures are high.

Over the oceans and the polar regions, where evaporation is typically at the potential rate, the two definitions coincide.

The potential evaporation rates computed by means of Budyko's definition and depicted in Figs. 2 and 3 would be different if they had been computed from a model simulation that actually employed Budyko's (4). In that case, the induced changes in evaporation would modify the atmospheric state, leading to further changes in the potential evaporation rate. Preliminary numerical experiments with the GFDL GCM, however, indicate that this feedback is small, particularly due to the relatively small effect on actual evaporation, as discussed below.

### 5. Control of GCM soil moisture by the water balance

Having established the existence of a substantial difference between the annual-average values of  $E_p(T_s)$

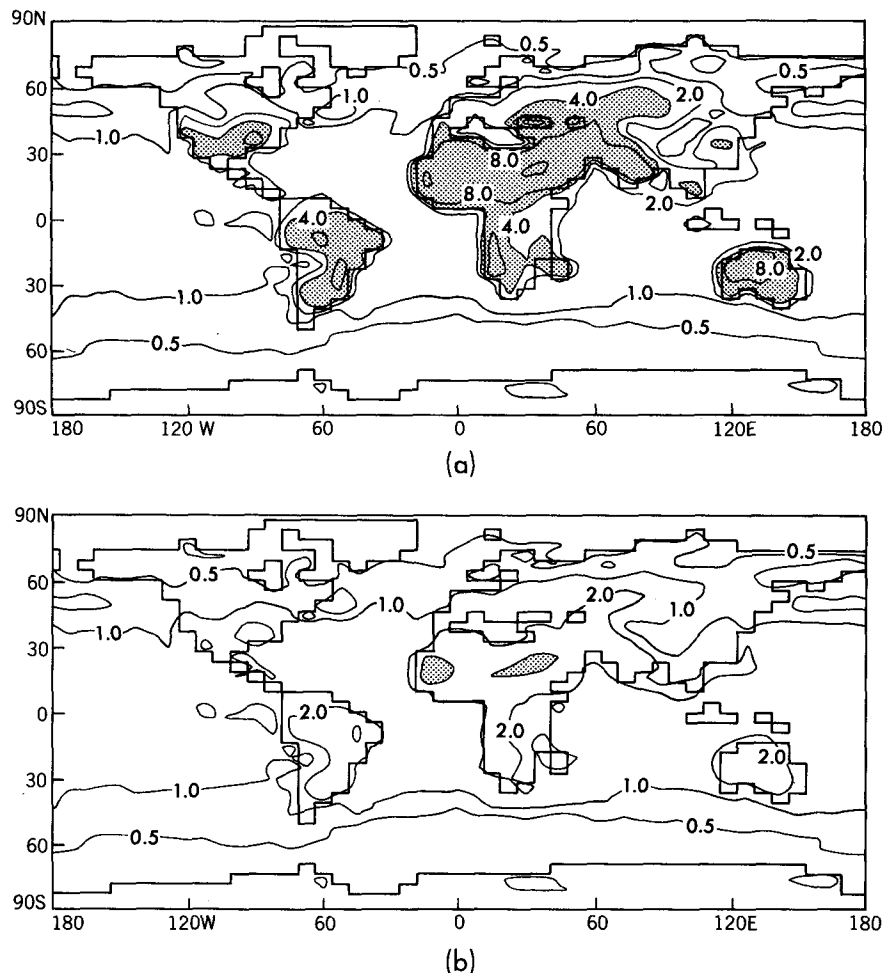


FIG. 2. Computed annual-average rates of potential evaporation ( $\text{m yr}^{-1}$ ); (a)  $E_p(T_s)$ , (b)  $E_p(T_w)$ .

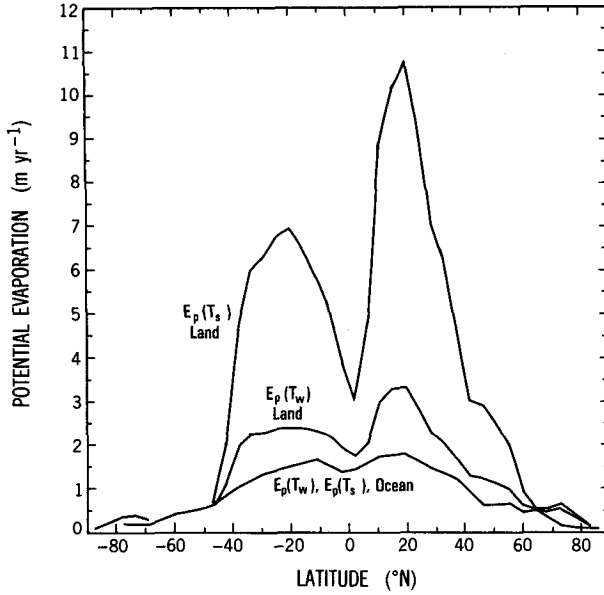


FIG. 3. Computed zonally and annually averaged rates of potential evaporation as function of latitude.

and  $E_p(T_w)$  over vast continental areas, it is natural to try to determine the impact of this difference on actual evaporation. The direct effect to be expected (Sud and Fennessy 1982; Sellers 1987) would be increased evaporation, given the assumed direct proportionality between potential and actual rates. However, increased evaporation reduces soil moisture, providing a negative feedback of considerable importance. Therefore, the sensitivity of actual evaporation to potential evaporation must be determined in the framework of a land-water balance. The water-balance equation for snow-free land points in the GCMs under consideration is of the form

$$\frac{dw}{dt} = P - E - Q = P - E_p \min\left(\frac{w}{w_k}, 1\right) - Q, \quad (24)$$

in which  $P$  is the precipitation intensity and  $Q$  is the runoff rate. In the GFDL model,  $Q$  is simply the excess water input after  $w$  reaches field capacity, while other models additionally include runoff, dependent on  $w$ , for unsaturated field capacity. In principle, then, the determination of the effect of potential evaporation on actual evaporation requires consideration of all terms in the water balance, and all times of the year are interrelated by the storage term. The dynamics of  $w$  is rather complicated, since it varies both seasonally, due to the seasonality of climate, and on a much shorter time scale, mainly due to the episodic delivery of precipitation by the atmosphere. It follows that (24) may be useful for a simulation analysis of the sensitivity problem, but it does not readily yield much in the way of physical insight regarding average conditions. The

problem of defining the control of soil moisture by the water balance becomes considerably simplified when one considers only the climatological average water balance and recognizes the relatively short memory of climatological-average soil moisture. For simplicity, only the case where  $w$  does not exceed  $w_k$  at any time will be considered. By taking an ensemble average of (24), we then obtain

$$\frac{d\bar{w}}{dt} = \bar{P} - \bar{E}_p\left(\frac{\bar{w}}{w_k}\right)(1 + R_{E_p,w}) - \bar{Q} \quad (25)$$

in which the overbar denotes an ensemble average and  $R_{E_p,w}$  is the covariance between  $E_p$  and  $w$ , normalized by the product of their means. Each quantity in (25) varies only on an annual time scale and can be approximated by the use of monthly or seasonal averages computed from climate models. Conversely, (25) provides a basis for the interpretation of the average fields typically examined in climatic modeling studies.

Consider now the water balance for a grid cell in the GFDL climate model during a period when  $w$  is less than  $w_k$  and snowmelt is absent. Because runoff occurs only when  $w$  is at the field capacity (which is greater than  $w_k$ ), the average water balance can be written simply as

$$\frac{d\bar{w}}{dt} = \bar{P} - \bar{E}_p\left(\frac{\bar{w}}{w_k}\right)(1 + R_{E_p,w}). \quad (26)$$

As  $\bar{P}$  and  $\bar{E}_p$  vary seasonally with the time scale of 12 months,  $\bar{w}$  adjusts to those variations on a time scale of  $w_k/[\bar{E}_p(1 + R_{E_p,w})]$ , which is essentially the evaporative damping time scale introduced by Delworth and Manabe (1988). If one uses  $E_p(T_w)$ , this time scale is on the order of 1 month all year in low latitudes and during the summer half-year in middle latitudes; it is even smaller if  $E_p(T_s)$  is used. The combination of slowly varying forcing and relatively fast response leads to a situation in which the (climatological average) soil moisture is close to equilibrium with the (climatological average) forcing at any time, and thus the storage term is small compared to the forcing term. Since runoff is essentially another damping term in (24), its inclusion does not materially change this argument, and in fact it leads to a further reduction in the memory of soil moisture. For the specific conditions leading to (26), the approximation is

$$\bar{P} \approx \bar{E}_p\left(\frac{\bar{w}}{w_k}\right)(1 + R_{E_p,w}). \quad (27)$$

In view of the preceding discussion, it appears useful also to introduce a more general, approximate expression of the water balance of land grid points in all of those GCMs employing (6):

$$\bar{P} - \bar{E} - \bar{Q} \approx 0. \quad (28)$$

Approximations (27) and (28), in which the storage terms are neglected, provide a simple, new framework for analyzing the relations among soil moisture and the various components of the average water balance. According to this result, the average value of soil moisture at a certain time during the year is determined mainly by interactions with the atmosphere at that time and is only minimally dependent on antecedent conditions—that is, the soil moisture has a short memory at the time scale of the annual cycle. For example, the soil moisture found in June, on average, is functionally related to the precipitation and potential evaporation rate during the typical June and is only weakly dependent on, say, soil moisture recharge the previous winter or spring.

The foregoing analysis did not explicitly account for, but neither is it inconsistent with, the dependence through atmospheric processes of  $P$  and  $E_p$  on  $w$ . As  $w$  varies through the year, it imposes variations upon  $E$  and thereby influences the seasonal cycles of  $P$  and  $E_p$ . Suppose that the relation between regional evaporation and regional precipitation is given by the very simple expression

$$P = P_0 + \alpha E, \quad (29)$$

in which  $P_0$  and  $\alpha$  parameterize the regional response of the atmosphere. For illustrative purposes, assume that  $E_p$  is unaffected by  $E$ , that  $\alpha$  is a positive constant less than unity, and that the grid-scale water balance equations apply also at the regional scale now considered. It can then be shown that the time scale of adjustment of the climatological average soil moisture to the seasonally varying forcing by  $P_0$  and  $E_p$  is given by  $w_k / [\bar{E}_p(1 + R_{E_p,w})(1 - \alpha)]$ . If, for example,  $\alpha$  is 0.5, then the time scale of adjustment of soil moisture to  $P_0$  and  $E_p$  is double its time scale of adjustment to  $\bar{P}$  and  $\bar{E}_p$ . Therefore, the consideration of atmospheric feedback exposes the longer memory of soil moisture; the use of  $\bar{P}$  as an independent variable disguised part of the true memory.

One may also consider the problem of adjustment of a gridpoint anomaly of soil moisture. Suppose, in analogy to (29), that departures from the climatological average behavior are described by

$$P - \bar{P} = \gamma(E - \bar{E}). \quad (30)$$

If departures of  $E_p$  from its climatological average are ignored, it can be shown that the time scale of decay of a gridpoint soil moisture anomaly is then given by  $w_k / [\bar{E}_p(1 + R_{E_p,w})(1 - \gamma)]$ . An analogous decay time scale has been estimated by Delworth and Manabe (1988), under the assumption that soil moisture is the output of a first-order Markov process, using both spectral and time-series analyses of output from a GCM. Their values of the decay time scale for equatorial and subtropical regions were approximately 1 month. Reported annual-average midlatitude values were about 2 months, probably reflecting values closer

to 1 month in summer and significantly longer in winter. Allowing for some departure of  $\gamma$  above zero and the crudeness of the present analysis, this is consistent with the earlier estimate that values of the time scale of adjustment of soil moisture in the absence of feedbacks are typically less than 1 month when  $E_p(T_s)$  is used with  $\beta_w$ .

## 6. Sensitivity of soil moisture to potential evaporation

The introduction of (27) and (28) greatly facilitates the analysis of the sensitivity of the water balance to the definition of potential evaporation. The greatest sensitivity is to be expected when and where the two definitions of potential evaporation differ—that is, under conditions of limited soil moisture. Under such conditions, runoff is negligible. Ignoring changes in precipitation and in  $R_{E_p,w}$ , (27) implies that the artificial increase in potential evaporation caused by use of  $E_p(T_s)$  must be offset by a decrease of soil moisture by the same factor. Then to the order of accuracy of (27), the evaporation rate remains unchanged, and evaporation is limited in the dry season by the dry-season precipitation.

However, there is a second-order effect on evaporation, associated with the neglected storage term in the water balance. In a seasonal climate the winter (or wet-season) soil moisture will be independent of the definition of potential evaporation, and the summer (or dry-season) soil moisture will be lower when  $E_p(T_s)$  is used in the simulations than when  $E_p(T_w)$  is used; so different definitions of potential evaporation will yield different amounts of storage depletion during the transition between seasons. It can be expected that  $E_p(T_s)$  will therefore allow somewhat more evaporation during the wet-dry transition. Correspondingly, as summer ends, there will be a larger storage deficit to fill and, therefore, somewhat less evaporation during the dry-wet transition. Since the soil will tend to saturate later in the season, runoff will begin later, and the net effect of using  $E_p(T_s)$  will be to decrease annual runoff and to increase annual evaporation. It follows from the foregoing that the amount of additional evaporation will not exceed the induced reduction in minimum soil moisture.

The order of magnitude of the reduction in minimum soil moisture can be estimated from (22) and (27). In particular, it can be shown using these equations that the largest possible reduction of minimum climatological soil moisture induced by use of  $E_p(T_s)$  occurs when the minimum soil moisture is about half of  $w_k$  and that that reduction is approximately

$$(w_k \zeta) / [4(1 + \zeta)].$$

For the GFDL climate model,  $w_k$  is ordinarily 11.25 cm. This implies that the reduction in summer soil moisture induced by use of  $E_p(T_s)$  has an upper bound



of about 2–3 cm. The associated increase in annual evaporation will be less. For comparison, the global average evaporation from land is about 60 cm in the climate model.

The interpretation of results by means of (27) is complicated somewhat by the fact that the values of  $R_{E_p, w}$  depend on the definition used for  $E_p$ . With (7) and (8) there is a strong, direct negative correlation between  $E_p$  and  $w$ , while it can be expected (and preliminary simulations with a GCM verify) that the correlation is relatively small when (1) and (3) are used. The result is that the factor by which average soil moisture is artificially reduced by employing  $E_p(T_s)$  in a GCM simulation will be slightly smaller than the factor by which potential evaporation is increased.

It seems desirable to supplement the foregoing approximate analysis with more detailed simulations. This is done using a simple land–surface hydrologic parameterization, driven by observed climatological statistics from diverse climates, without consideration of atmospheric feedbacks. The parameterization is essentially equivalent to the formulation of Manabe (1969), except that a time step of one day is employed and freezing conditions are ignored. Forcing variables include daily average net incoming radiation and precipitation, as well as air temperature, specific humidity, and wind speed, each at measurement level. (The wind speed,  $u$ , enters through  $r_a$ , taken as the reciprocal of  $0.003u$  in the GFDL model.)

The forcing variables are prescribed on the basis of observed climatologies in the case of precipitation, temperature, humidity, and wind. The incoming shortwave and longwave irradiances are estimated from standard empirical relations that use surface measurements of sky cover, sunshine duration, air temperature, and humidity. Climatological data were obtained from Rudloff (1981), Court (1974), and Ratisbona (1976). For all variables except precipitation, the daily values were set equal to the relevant monthly mean. Occurrence of precipitation was modeled as a Poisson process on a daily time scale, and storm depths were taken to be gamma distributed. The event probabilities and the mean storm depths were estimated from the available data so as to preserve the reported mean monthly precipitation and the reported mean numbers of days per month with total precipitation above a given threshold. The shape parameter of the gamma distribution was varied within reasonable bounds, and it was found that the sensitivity to its value was small; so the results reported here are for a value of unity for which the gamma distribution reduces to the exponential distribution.

Table 1 summarizes the results of a series of simulations for eight different locations, ranging from cool and wet to hot and dry. For each station, the summary is given for annual total fluxes and for average fluxes during the month with the lowest soil moisture. The aridity of the sites is characterized by the radiative index

of dryness (Budyko 1974, p. 322), which is the ratio of annual mean net radiation to the amount of energy that would be required to vaporize the annual mean precipitation. Values less than unity indicate a relative abundance of water, while larger values indicate water shortage. The annual average values of potential evaporation, computed by the two methods, are shown in Table 1. Consistent with the approximate analysis, the relative difference between the two definitions increases with the aridity. Annual-average values of potential evaporation are amplified by factors near 1.5 in humid climates and as large as 3 in dry climates. Amplification factors during the driest month range from 2 to more than 3.

Also shown in Table 1 are the annual and driest-month totals for actual evaporation, normalized by precipitation for the same period. Under moderately humid conditions (the first five cases), the change in annual potential evaporation rate is translated into very small (1%–2%) changes in the annual actual evaporation, and the signs and magnitudes of those changes are consistent with the results of the previous section. Under drier conditions, the soil is never saturated, runoff never occurs, and hence there is no way that the annual evaporation can change. Under continuously humid conditions, it can be expected that the two definitions of potential evaporation would always coincide, and so again there would be no sensitivity of the annual total evaporation. Actual evaporation during the driest month is approximately equal to the precipitation during that month; this is a result both of the short memory of soil moisture and of choosing a month for which  $w$  is a minimum and, hence, the storage rate changes sign.

Also listed in Table 1 are the values of annual average and driest-month average soil moisture associated with each simulation. Soil moisture is normalized by the field capacity,  $w_0$ , which is set to 15 cm in agreement with Manabe (1969). In each case the higher values of potential evaporation lead to lower values of soil moisture, and the relative change of soil moisture is greatest under the most arid conditions. For the individual months, the amplification of potential evaporation induced by use of  $E_p(T_s)$  is almost equaled by the reduction of soil moisture; the slight discrepancy can be attributed to differences in  $R_{E_p, w}$ , as discussed earlier. Dry-season soil moisture is typically underestimated by a factor between 2 and 3 under all climatic conditions. Consistent with the estimate made at the beginning of this section, the largest differences in minimum soil moisture are about 2 cm. Clearly the simulated values of soil moisture are much more sensitive to the definition of potential evaporation, in a relative sense, than are the values of actual evaporation.

The slight excess annual evaporation resulting from use of  $E_p(T_s)$  leads to slightly lower annual-average surface temperatures, as shown in Table 1. The cooling is on the order of 0.1°C. For the months with the lowest

TABLE 1. Summary of annual and driest-month land-surface water and energy balances for several locations using hydrologic parameterization of Manabe (1969) in conjunction with observed climatological data:  $R$ —net radiation;  $L$ —latent heat of evaporation;  $P$ —precipitation;  $E_p$ —potential evaporation;  $E$ —evaporation;  $w$ —soil moisture;  $w_0$ —field capacity;  $T_s$ —surface (“skin”) temperature. Overbar denotes time average. (1) Standard parameterization using  $E_p(T_s)$ . (2) Potential evaporation replaced by  $E_p(T_w)$  in parameterization.

Location		$\frac{\bar{R}}{\bar{L}\bar{P}}$	$\frac{\bar{E}_p}{\bar{P}}$		$\frac{\bar{E}}{\bar{P}}$		$\bar{w}/w_0$		$T_s$
		(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1) - (2)
Vancouver, Canada	Annual	0.80	1.2	0.79	0.50	0.49	0.64	0.69	0.07
	August	3.0	7.2	3.3	0.85	0.90	0.097	0.21	-0.06
Santarem, Brazil	Annual	0.75	1.1	0.72	0.57	0.56	0.67	0.73	0.15
	October	2.7	8.8	3.4	1.2	1.2	0.11	0.26	-0.04
Manaus, Brazil	Annual	0.78	1.2	0.76	0.60	0.59	0.66	0.72	0.17
	September	2.0	7.0	2.6	0.89	0.89	0.12	0.26	-0.04
Cuiaba, Brazil	Annual	1.0	1.6	1.0	0.77	0.75	0.58	0.63	0.16
	August	6.5	28.	11.	1.6	2.0	0.046	0.14	-0.35
Washington, DC	Annual	1.2	2.6	1.5	0.97	0.96	0.44	0.55	0.05
	July	1.6	5.0	2.1	0.98	0.95	0.18	0.34	0.12
San Antonio, Texas	Annual	2.0	7.0	2.8	1.0	1.0	0.17	0.29	0.01
	August	2.4	13.	3.9	0.91	0.89	0.073	0.17	0.05
Abilene, Texas	Annual	2.3	10.	4.0	1.0	1.0	0.11	0.20	0.01
	August	3.9	26.	7.5	1.1	1.2	0.039	0.12	-0.16
Phoenix, Arizona	Annual	4.5	33.	11.	1.0	1.0	0.055	0.094	0.01
	June	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	<.001	0.002	-0.05

soil moisture, the differences in surface temperature are somewhat random, following the differences in monthly evaporation rates, which have no consistent sign. (As noted earlier in this section, the evaporation difference changes sign near the time when soil moisture reaches its minimum. This may occur at any time during the month, so the monthly evaporation difference can have either sign.) The months with the maximum differences in temperature (not shown in the table) are those during the wet-to-dry transition, when the extra evaporation mainly occurs. Then the temperatures differ by  $0.25^\circ$ – $1.0^\circ\text{C}$ .

In this section the sensitivity of actual evaporation to the definition of potential evaporation is considered only for the case of the surface hydrologic parameterization in the GFDL GCM. A distinctive feature of the GFDL GCM is that it allows runoff to occur only when the soil moisture has reached field capacity. In reality, land produces runoff under conditions of less-than-maximum soil moisture due to several physical processes, including limitations on infiltration rates due to insufficient soil permeability, localized saturation of the soil in poorly drained areas, spatial variability of precipitation and soil moisture, and natural gravity-induced drainage of land. In recognition of this, many other GCMs have incorporated various empirical or conceptual schemes for the production of extra runoff (Carson 1982). In each case, the amount of runoff generated under a given set of conditions increases with the soil moisture. The significance of this for the current discussion is that the sensitivity of runoff to soil mois-

ture in such models sensitizes their dry-season evaporation rates to the definition of potential evaporation. This can be seen by considering a special case of (24) in the form

$$\frac{dw}{dt} = P - E_p\left(\frac{w}{w_k}\right) - \mu\left(\frac{w}{w_0}\right)P, \quad (31)$$

in which  $\mu$  is a dimensionless runoff coefficient, and  $w_0$  is the field capacity. Taking an ensemble average, and neglecting the storage term, we transform (31) to

$$\bar{P} \approx \frac{\bar{E}_p}{w_k} (1 + R_{E_p,w}) \bar{w} + \frac{\mu \bar{P}}{w_0} (1 + R_{P,w}) \bar{w}, \quad (32)$$

in which  $R_{P,w}$  is the covariance between precipitation and soil moisture, normalized by the product of their means. Equation (32) can be solved for the average soil moisture,

$$\bar{w} \approx \left( \frac{\mu}{w_0} (1 + R_{P,w}) + \frac{1}{w_k} (1 + R_{E_p,w}) \frac{\bar{E}_p}{\bar{P}} \right)^{-1}. \quad (33)$$

The resulting rate of evaporation is

$$\bar{E} \approx \bar{P} \left( 1 + \frac{\mu w_k}{w_0} \frac{(1 + R_{P,w})}{(1 + R_{E_p,w})} \frac{\bar{P}}{\bar{E}_p} \right)^{-1}. \quad (34)$$

In the dry season, the introduction of an artificially high value of  $E_p$  will thus still tend to drive  $w$  down. However, the lower  $\bar{w}$  will reduce the magnitude of the runoff term and hence lead to an artificial increase in actual evaporation. For the GFDL GCM,  $\mu$  is zero and

the value of dry-season  $E$  is insensitive to the value of  $E_p$ , as discussed earlier. However, for a nonzero  $\mu$ , the actual  $E$  becomes directly dependent upon  $E_p$ , as expressed by (34).

The water- and energy-balance simulation experiment described earlier was repeated using the additional runoff implicit in (31). Values of  $\mu$  were assigned geographically, following the suggestions of Zubenok (1978), and ranged from 0.2 to 0.6. The results are given in Table 2 for the same sites as those in Table 1. In all cases, the evaporation totals are lower than those in Table 1 because of the greater tendency of the modified scheme to produce runoff. (The decreased evaporation leads to somewhat higher surface temperatures, hence higher terrestrial radiation and lower values of the radiative index of dryness.) The sensitivities of actual evaporation to the definition of the potential rate are considerably larger than before, and this can be attributed to the effect described above. Soil moisture values in Table 2 are consistently less than those in Table 1, but their sensitivities to the  $E_p$  definition are comparable. The differences in annual-average surface temperature are larger in Table 2 than in Table 1, and this corresponds directly to the larger differences in evaporation.

**7. Implications for climate modeling**

The parameterization of evaporation given by (6), with  $\beta_s$  replaced by  $\beta_w$ , has been used widely in GCM-based studies of climate and climatic change. In view of the systematic difference between  $\beta_w$  and  $\beta_s$ , it is

important to consider the possible implications for such studies. It has been noted, in our analyses without atmospheric feedbacks, that the largest errors are in the computed values of potential evaporation, which have been greatly overestimated. Also substantially affected are the values of soil moisture, which have been significantly underestimated. In Manabe's (1969) parameterization of land-surface hydrology, the soil moisture reduction almost exactly compensates for the increase in potential evaporation, and it follows that the annual partitioning of precipitation between evaporation and runoff is only slightly changed. However, this low sensitivity is a result of the lack of any direct dependence of runoff upon soil moisture under unsaturated conditions, and such independence may be excessively unrealistic. In models that include a significant sensitivity of runoff to soil moisture, it can be expected that actual evaporation rates would be more substantially affected. It should also be kept in mind that the atmospheric reaction to the differences in evaporation could lead, through various feedback loops, to further modifications in the evaporation rates. For instance, extra evaporation would lead to both extra precipitation and extra cloud cover, which might, respectively, enhance and suppress evaporation.

It must be acknowledged that the tendency toward excessive evaporation, resulting from the use of an inconsistent combination of  $\beta$  and potential evaporation in GCM-based climate models, does not necessarily result in excessive modeled evaporation relative to the real world. In the first place, it is possible that errors in computed forcing of the land surface by the atmo-

TABLE 2. Same as Table 1, but with Manabe's (1969) parameterization modified to produce additional runoff at rate  $\mu(w/w_0)P$  when soil moisture is less than field capacity. (a) Standard parameterization using  $E_p(T_s)$ . (b) Potential evaporation replaced by  $E_p(T_w)$  in parameterization.

Location		$\frac{\bar{R}}{L\bar{P}}$	$\frac{\bar{E}_p}{\bar{P}}$		$\frac{\bar{E}}{\bar{P}}$		$\bar{w}/w_0$		$T_s$
		(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1) - (2)
Vancouver, Canada	Annual	0.80	1.3	0.79	0.48	0.46	0.62	0.67	0.08
	August	3.0	7.2	3.3	0.83	0.85	0.093	0.20	-0.03
Santarem, Brazil	Annual	0.72	1.2	0.72	0.52	0.50	0.56	0.62	0.32
	October	2.6	9.4	3.4	1.0	0.96	0.087	0.21	0.09
Manaus, Brazil	Annual	0.75	1.3	0.75	0.55	0.52	0.52	0.59	0.39
	September	1.9	7.4	2.5	0.81	0.73	0.098	0.22	0.28
Cuiaba, Brazil	Annual	0.96	1.8	1.0	0.67	0.63	0.38	0.46	0.42
	August	6.1	30.	11.	1.1	1.5	0.029	0.10	-0.37
Washington, D.C.	Annual	1.2	2.8	1.5	0.85	0.81	0.34	0.45	0.17
	June	2.1	6.1	2.6	1.1	1.0	0.15	0.29	0.24
San Antonio, Texas	Annual	2.0	7.2	2.8	0.92	0.88	0.14	0.25	0.12
	August	2.4	13.	3.9	0.86	0.81	0.066	0.16	0.15
Abilene, Texas	Annual	2.3	10.	4.0	0.97	0.95	0.10	0.19	0.04
	August	3.9	26.	7.5	1.0	1.1	0.038	0.11	-0.12
Phoenix, Arizona	Annual	4.4	33.	11.	0.98	0.97	0.053	0.091	0.03
	June	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	<0.001	0.002	-0.05

sphere may offset (or compound) any error in the evaporation model. Furthermore, it is possible to compensate for the noted idiosyncrasy of the parameterization of evaporation by means of calibration, that is, by biasing some other aspect of the continental hydrologic parameterization in such a way as to match observations. For example, Hansen et al. (1983) were able to vary widely the annual global runoff from a GCM by adjusting the value of a runoff parameter similar to  $\mu$  in (31). Of course, such an approach can only tune the annual water balance easily and will not necessarily correct systematic distortions of seasonality of the water balance. In addition, it does not resolve the discrepancy between modeled and observable soil moisture, nor that between definitions of potential evaporation.

To the extent that evaporation from the continents is overestimated (and runoff is underestimated) by the land-surface hydrologic parameterization, it can be expected that atmospheric (and oceanic, if coupled) states in a climate model will be biased as a result. Numerical experiments, with the climate models, are needed to assess the magnitudes of these sensitivities; their consideration is beyond the scope of this paper.

#### a. Potential evaporation rates

This paper may help to explain certain peculiar results of previous GCM-based studies of climate, particularly those results relating directly to potential evaporation or to soil moisture. Delworth and Manabe (1988) noted significant differences between their GCM-computed potential evaporation rates and those calculated using a very rough approximation of  $E_p(T_w)$ . The GCM potential evaporation rates reported by Delworth and Manabe for southern North America have averages mostly in the range from 1 to 3 cm d<sup>-1</sup> for the period June through August. This implies annual total potential evaporation significantly in excess of 1–3 m, perhaps double these values. Similarly, Rind et al. (1990) reported a summer-average potential evaporation rate for the same area of 3.35 cm d<sup>-1</sup>. This suggests an annual total of much more than 3 m. Figure 2a shows annual totals in excess of 3 m over much of the same area. In contrast to these GCM estimates, the reported mean annual lake evaporation (roughly equivalent to potential evaporation) for this region is mostly in the range from 0.7 to 1.5 m (U.S. Dept. of Commerce 1968). Note that these lake evaporation figures agree much more closely, as expected, with the values of  $E_p(T_w)$  in Fig. 2b, which fall mostly in the range of 1–2 m within this area. Additionally, Korzun (1974) provides maps of potential evaporation that specify values in the range 0.8–1.8 m for the same area. Thus, the distinction made between  $E_p(T_w)$  and  $E_p(T_s)$  in this paper reconciles the strong divergence between the estimates of potential evaporation made by GCMs and those derived from observations by means of conventional hydrologic practice.

#### b. Soil moisture

Several studies have focused on global distributions of soil moisture calculated by GCM-based climate models. However, due to the difficulty of collecting and systematically analyzing data on soil moisture at scales relevant for GCMs, it has been impossible to perform direct comparisons between GCM estimates and observations. Recently, though, Vinnikov and Eslerkepova (1989) analyzed the measurements of moisture in the top meter of soil made over periods of at least 10 years under natural vegetation at 50 sites in the western Soviet Union. They also compared their observations to nearest-neighbor gridpoint values obtained by Manabe and Wetherald (1987) and by M. E. Schlesinger (personal communication). They found that the GCM estimates of summer soil moisture were consistently much lower than the observations, but that the two spatial patterns were well correlated. Typical August values produced by the models were less than 5% of field capacity, while observations were typically in the range from 25% to 75% of field capacity. It is thus clear that the observed soil moisture exceeds the modeled values throughout the area, typically by a factor of about 10. In view of the argument set forth earlier, it would appear that this discrepancy can be attributed partially to the excessive values of potential evaporation. For August in this region, computations similar to those for Fig. 2 show that the GCM overestimates the potential evaporation by a factor of 2 to 4 due to the use of  $E_p(T_s)$  in place of  $E_p(T_w)$ . Based on the computations reported in the previous section, this translates to a diminution of soil moisture by a factor of about two to three. It appears that the remaining factor of about 3 to 5 may be attributed, by means of (27), to the very low computed precipitation in the GCM in this region in summer.

It may be anticipated that the difference in definitions of potential evaporation will have implications for the time scale of decay of soil moisture anomalies and hence for variability of soil moisture (Delworth and Manabe 1988) and of the atmosphere (Delworth and Manabe 1989). Lower values of potential evaporation should give rise to longer time scales of decay of anomalies of soil moisture and associated variables. It follows that the persistence of soil moisture anomalies may have been underestimated by Delworth and Manabe (1988). Furthermore, the importance of soil moisture for variability of the atmosphere may have been understated by Delworth and Manabe (1989). It is not possible to give a simple, quantitative estimate of the magnitudes of these changes here, since they will undoubtedly be influenced by atmospheric feedbacks. However, to the extent that these features of the climate model are sensitive to the definition of potential evaporation, this paper underscores the importance of the processes identified in those studies.

### c. Summer dryness

Much of the attention given to GCM-derived soil moisture recently in the literature has centered on the implications of enhanced greenhouse warming for seasonal-average soil moisture. Manabe et al. (1981) found that modeled equilibrium increases of atmospheric carbon dioxide induced reductions of soil moisture in middle and high latitudes during the summer. Some subsequent studies have supported the earlier results, but generally there have been significant points of disagreement even on the sign of the soil moisture change. A comparison of the results of five GCM-based climate models has been presented by Kellogg and Zhao (1988). Apparently all five models employed a scheme similar to (6), with  $\beta_s$  replaced by  $\beta_w$ , in order to calculate continental evaporation; so it can be expected that in each case the control simulation (natural carbon dioxide concentration) will tend to yield excessively dry soil in summer.

Equation (33) may shed some light on the factors controlling changes in summer dryness induced by greenhouse warming. If the changes in correlation are negligible, then changes in  $\bar{w}$  result only from changes in  $\bar{P}$  and  $\bar{E}_p$ . For small changes, differentiation of (33) leads to

$$\frac{d\bar{w}}{\bar{w}} \approx \left[ 1 + \frac{\mu w_k (1 + R_{p,w}) \bar{P}}{w_0 (1 + R_{E_p,w}) \bar{E}_p} \right]^{-1} \left( \frac{d\bar{P}}{\bar{P}} - \frac{d\bar{E}_p}{\bar{E}_p} \right). \quad (35)$$

This implies that the magnitudes of changes in soil moisture are directly proportional to the value of soil moisture in the control experiment and to the difference between the relative increase of precipitation and the relative increase of potential evaporation. Furthermore, the introduction of a nonzero value for the runoff coefficient  $\mu$  (or, in general, the allowance for runoff under unsaturated conditions) will tend to dampen the change in soil moisture. The sign of the soil moisture change is determined by the relative magnitudes of the relative changes in precipitation and potential evaporation. It can be concluded from (35) that predicted changes in dry-season soil moisture associated with greenhouse warming are highly dependent on the control experiment, that they are tied to dry-season changes in precipitation and potential evaporation, and that they may differ due to differences in the parameterization of runoff.

It has been suggested herein that seasonal memory is not an important factor in the interpretation of the annual cycle of climatological-average soil moisture. This is in contrast to some other past studies, in which seasonal memory has been presumed, implicitly or explicitly, to play a more significant role. Manabe and Wetherald (1987) attributed part of the simulated summer dryness to the earlier snowmelt season and the earlier cessation of late-spring rainfall of the doubled- $\text{CO}_2$  climate. Similarly, Mitchell and Warrilow (1987) cited the earlier snowmelt season as one cause

of summer dryness. Further analyses will be needed to define in a more quantitative way the factors controlling summer dryness.

It might be suspected that the past predictions of summer dryness have been severely distorted by the use of  $E_p(T_s)$ . It is possible to evaluate this question in a rather crude way by making use of some of the results already presented here. Consider the hydrologic parameterization in the GFDL GCM. Suppose, as has been suggested herein, that the use of different definitions of potential evaporation in that model would result in significant changes only in soil moisture and potential evaporation and that the atmospheric sensitivity is minimal. Then (22) can be employed to relate the two values of potential evaporation that would occur in two experiments employing the two different definitions. As a rough approximation, it may be assumed that (22) holds for the ensemble average values of the constituent variables. Then

$$\frac{\overline{dE_p(T_s)}}{E_p(T_s)} = \frac{\overline{dE_p(T_w)}}{E_p(T_w)} - \frac{\bar{\xi} \bar{w}}{w_k} \frac{d\bar{w}}{\bar{w}} \quad (36)$$

The greenhouse-induced soil moisture change in the  $E_p(T_w)$  experiment is given by (35), with  $\mu$  equal to zero, as

$$\frac{d\bar{w}}{\bar{w}} \approx \frac{d\bar{P}}{\bar{P}} - \frac{\overline{dE_p(T_w)}}{E_p(T_w)}, \quad (37)$$

whereas the change in the  $E_p(T_s)$  experiment would be given approximately by the combination of (35) and (36) as

$$\frac{d\bar{w}}{\bar{w}} \approx \left( \frac{d\bar{P}}{\bar{P}} - \frac{\overline{dE_p(T_w)}}{E_p(T_w)} \right) \left( 1 + \bar{\xi} \frac{\bar{w}}{w_k} \right). \quad (38)$$

In the dry season of interest, the factor  $[1 + \bar{\xi}(\bar{w}/w_k)]$  is usually not far from unity, since  $\bar{\xi}$  is 2 or 3 and  $\bar{w}$  is rather low; in the extreme the relative soil moisture change may be doubled by the use of  $E_p(T_s)$ , but generally the amplification would be less. Qualitatively the drying should be similar. It should also be recalled that the actual magnitudes of changes will tend to be larger in the  $E_p(T_w)$  experiment since the control soil moisture will be higher. It can be tentatively concluded that the use of  $E_p(T_s)$  in the GFDL GCM has not significantly overstated the magnitude of summer dryness induced by greenhouse warming. A definite conclusion would require that the experiments on summer dryness be repeated using (4).

## 8. Summary

It is possible to model evaporation empirically as the product of a potential evaporation rate,  $E_p$ , and a moisture availability function,  $\beta$ . The approach of Bu-

dyko (1956) was to use the product  $\beta_w E_p(T_w)$ , in which  $E_p(T_w)$  is the rate of evaporation that would be realized if the surface were completely wet and if the incident radiation and atmospheric state were held constant. The corresponding moisture availability function,  $\beta_w$ , has been estimated experimentally in many field studies. The approach of many climate studies with GCMs is to use the product  $\beta_s E_p(T_s)$ , in which  $E_p(T_s)$  is the rate of evaporation that would be realized if the surface were completely wet and if the surface temperature were held constant. It can be shown that  $E_p(T_s)$  is typically much larger than  $E_p(T_w)$  when soil moisture is limited, so  $\beta_s$  should then be considerably smaller than  $\beta_w$  for a given level of soil moisture. However, the standard practice in climate modeling studies has been to use functions  $\beta_s$  that approximate, or even exceed, empirically determined  $\beta_w$  functions. The net effect is that many GCMs will tend to overestimate significantly the rate of evaporation that corresponds to a given level of soil moisture.

A brief consideration of the problem of subgrid variability of soil moisture gives further support to the conclusion stated above, since the grid-scale  $\beta_w$  function can be shown to lie near or below the empirically determined subgrid function.

Some simple physical analyses provided in the Appendix suggest that neither the  $\beta_w E_p(T_w)$  nor the  $\beta_s E_p(T_s)$  approach is very consistent with the physics of small land areas, since in either case it can be shown that the  $\beta$  function must depend on factors other than soil moisture. However, in the case of large subgrid variability of soil moisture, it can be shown that  $\beta_w$  can be approximated by a unique function of soil moisture.

The dry-season memory of climatological-average soil moisture in GCMs, and probably in nature, is typically shorter than the time scale of the seasonal variation of climatic forcing. This means that the storage term may often be ignored, to first order, in approximate water-balance analyses of GCM soil moisture climatologies. In the present study, it leads to the conclusions, for the GFDL GCM, that dry-season evaporation is equal, to first order, to dry-season precipitation and that the main impact of the combined use of  $E_p(T_s)$  and  $\beta_w$  in the GFDL GCM is probably the artificial reduction of dry-season soil moisture. In GCMs that produce runoff under unsaturated conditions, the combined use of  $E_p(T_s)$  and  $\beta_w$  can be expected to produce similar effects and, in the long run, to induce excessive evaporation at the expense of reduced runoff, unless an appropriate bias is introduced to compensate for this effect.

This paper helps to explain some differences between GCM-generated and observation-based estimates of potential evaporation rates and soil moisture. The excessive rates of potential evaporation that have been noted in some GCMs can be attributed to the difference in the definitions explored in this paper. The unreal-

istically low values of dry-season soil moisture that have been noted in some GCMs are attributable, at least in part, to the problem with potential evaporation.

Implications for past analyses of greenhouse warming are unclear. Quantitative GCM-derived predictions of summer dryness induced by greenhouse warming are probably distorted by the use of an inappropriate definition for potential evaporation, but a simple analysis suggests that the basic dependence of drying on atmospheric forcing has at least been qualitatively preserved in the studies. Further investigations could clarify this issue.

It is possible to modify existing GCM-based climate models to remove the inconsistency noted in this paper. One approach is to carry a second surface energy balance equation for the hypothetical wet-surface temperature and to model evaporation as  $\beta_w E_p(T_w)$ . A second approach is to calculate  $\beta_s$  from  $\beta_w$  and the linear approximation of  $\zeta$ , using results presented here, and to model evaporation as  $\beta_s E_p(T_s)$ . Either approach should improve GCM estimates of potential evaporation and soil moisture.

It could also be noted that newer, physically oriented parameterizations of evaporation avoid this problem entirely. When they have been adequately validated and calibrated, they may provide a more satisfactory solution to the problem of modeling land-surface evaporation in GCMs, particularly for studies concerned directly with land-surface processes. In the meantime, the present paper offers a simple correction to the empirical approach, as well as assistance in the interpretation of past studies.

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## APPENDIX

### Physical Considerations

#### 1. Introduction

The main thrust of this paper concerns the implementation in GCMs of an empirically based parameterization of the evaporation process. As such, the major concern is with the mathematical manipulation of the relevant equations and the extraction of inferences regarding sensitivities of related variables. Nevertheless,

it is of some interest to consider the physical aspects of the differences between Budyko's (1956) parameterization and its implementation in GCMs. Is one of them fundamentally more representative of the physics than the other? From a practical standpoint, the empirical moisture availability function is available only for Budyko's formulation, but can Budyko's formulation be shown, in principle, to be superior or inferior to Manabe's (1969) formulation (with a consistently identified  $\beta_s$  function) on physical grounds? These questions are addressed in this Appendix.

It could be argued that the use of (4) is undesirable because it requires the introduction of a new temperature that does not, in general, correspond to any actual physical temperature in the climate system (although it is a well-defined function of other physical variables). The surface is then characterized by two temperatures—the actual temperature,  $T_s$ , and a hypothetical temperature,  $T_w$ , corresponding to free evaporation from the surface. Sellers (1987) points to the non-physical nature of  $T_w$  and quotes Y. Mintz, stating “in nature, there are no telephonic connections between lysimeters and the rest of the land surface”—that is, evaporation is not driven by the hypothetical temperature  $T_w$ . Indeed there is a strong case for questioning the form of (4) on physical grounds, but it is not apparent that a case has been put forward for the superiority of (6). One might also make the point that it is easier to solve one energy balance equation than two, but again the argument has no physical relevance.

Traditionally, the potential evaporation rate has been defined in such a way that it is minimally dependent on the actual rate of evaporation, representing in some sense the atmospheric demand for evaporation (but not necessarily allowing for atmospheric feedbacks if that rate were to be realized). The rate  $E_p(T_w)$  fulfills this criterion, but  $E_p(T_s)$  does not. In this sense,  $E_p(T_w)$  might be preferred over  $E_p(T_s)$ ; the former is quite robust, while the latter is extremely sensitive to the actual evaporation rate.

The physical significance and robustness of the variables discussed above is of interest, but such arguments do little to support the physical validity of parameterizing evaporation using either (4) or (6), since in either case the  $\beta$  function is still defined empirically. It is helpful to redefine the questions posed at the start of this Appendix by considering the stability of the  $\beta$  function. Since  $\beta$  is invariably expressed as a unique function of soil moisture in practice, it is suggested that the superior empirical formulation for evaporation would be the one that exhibits the least scatter in the relation between empirically determined values of soil moisture and  $\beta$ . Lacking the field data needed to approach the question in this way, we may instead approach it theoretically. For which formulation does a theoretical analysis, based on current understanding of the physics, suggest a better correlation between  $\beta$  and

soil moisture, with minimal dependence on other factors?

It is difficult to answer the question posed above definitively, since there is currently little agreement on a physically based parameterization of evaporation at GCM grid scale. Some investigators emphasize the importance of biophysical controls on evaporation, while others focus on the hydraulic resistance of the soil. Some treat the grid cell as a horizontally homogeneous domain, while others feel that heterogeneity itself is an important control. This situation is a sign of the complexity of the physical problem and the apparent lack of a consensus even on a framework for parameterization (unless that framework is defined simply as the union of all current attacks on the problem). The treatment given here will consider various approximate descriptions of limiting cases of the physical system.

## 2. Vegetation canopy under stomatal control

Consider first the case of a homogeneous, “big-leaf” vegetation canopy completely shading the ground (Monteith 1965). Then  $T_s$  represents the temperature of the leaf. Evaporation is assumed to occur inside the moist substomatal cavities of the leaf, and vapor diffusion out of the leaf is impeded by a bulk stomatal resistance  $r_s$  acting in series with the aerodynamic resistance  $r_a$ :

$$E = \frac{\rho}{r_a + r_s} [q_s(T_s) - q_a]. \quad (\text{A1})$$

As Brutsaert (1986) has noted, this conceptual model is consistent with the form of Manabe's (1969) parameterization, given here as (6) and (7), if the reduction factor is given by

$$\beta_s = \left(1 + \frac{r_s}{r_a}\right)^{-1}. \quad (\text{A2})$$

When (A1) is used to describe evaporation, the actual surface energy balance equation is

$$\begin{aligned} R_0 - 4\epsilon\sigma T_a^3(T_s - T_a) - G \\ = \frac{L\rho}{r_a + r_s} [q_s(T_s) - q_a] + \frac{\rho c_p}{r_a} (T_s - T_a). \end{aligned} \quad (\text{A3})$$

Substitution of (12), with  $T$  equal to  $T_s$ , into (A3), and subsequent solution, yield

$$T_s = T_a + \frac{R_0 - G - \frac{L\rho}{r_a + r_s} [q_s(T_a) - q_a]}{4\epsilon\sigma T_a^3 + \frac{\rho c_p}{r_a} + \frac{L\rho}{r_a + r_s} q'_s(T_a)}. \quad (\text{A4})$$

When this is substituted back into (A1), again using (12), it yields the evaporation rate

$$E = \frac{q'_s(T_a)(R_0 - G) + [4\epsilon\sigma T_a^3 + (\rho c_p/r_a)][q_s(T_a) - q_a]}{Lq'_s(T_a) + c_p(1 + r_s/r_a) + [4\epsilon\sigma T_a^3(r_a + r_s)]/\rho} \quad (\text{A5})$$

Substitution of (14) into (4) and comparison with (A5) show that Budyko's (1956) parameterization is consistent with the big-leaf model if and only if

$$\beta_w = \left\{ 1 + \frac{r_s}{r_a} \left[ \frac{\gamma \left( 1 + \frac{4\epsilon\sigma T_a^3 r_a}{\rho c_p} \right)}{\Delta + \gamma \left( 1 + \frac{4\epsilon\sigma T_a^3 r_a}{\rho c_p} \right)} \right] \right\}^{-1} \quad (\text{A6})$$

where use has been made of (19), (20), and (21).

It is conventionally assumed that stomata respond in a very complex way to many factors, including leaf water potential, leaf temperature, vapor pressure deficit of the air, and incident shortwave irradiance (Jarvis 1976). The relation with soil moisture is considered to be indirect, resulting from the coupling between soil moisture and the plant. In general, lower soil moisture restricts the entry of water into the root system of the plant and thus increases the likelihood of occurrence of leaf water potentials sufficiently low to induce closure of the stomata. In this sense, one can see from (A6) why  $\beta_w$  is found empirically to approach zero as soil moisture decreases and why it is insensitive to soil moisture when the soil moisture is high; the same limiting behavior can be expected of  $\beta_s$ .

Does either (A2) or (A6) describe a moisture availability function that is better correlated with soil moisture than the other? There is no clear answer. Superficially, (A2) appears simpler than (A6), but there is no reason to expect a better correlation with soil moisture since the stomatal resistance is not a unique function of soil moisture but rather depends in part on the additional variables entering (A6). When the problem is addressed in this framework of stomatal resistance, there is no clear physical basis for preferring the form of (4) over that of (6), or vice versa.

### 3. Vegetation canopy under soil-vegetal control

One may alternatively take the view that the stomatal resistance is more a result of control of evaporation by the plant than a cause of it. Cowan (1965) suggested that the rate of transpiration by a plant stand would be the lesser of a potential transpiration rate,  $E(r_{s,\min})$ , defined in a way analogous to (A5) with  $r_s$  equal to  $r_{s,\min}$  (the value of  $r_s$  when leaves are turgid) and a supply function,  $E_{sv}$ , dependent on the ability of the soil-vegetation system to deliver water to the substomatal cavities. The latter quantity depends on the soil moisture and relatively stable properties of the soil and the vegetation. Thus,

$$E = \min[E(r_{s,\min}), E_{sv}(w)]. \quad (\text{A7})$$

Combination of (4) and (A7) then gives

$$\beta_w = \min \left[ \frac{E(r_{s,\min})}{E_p(T_w)}, \frac{E_{sv}(w)}{E_p(T_w)} \right], \quad (\text{A8})$$

and combination of (6) and (A7) gives

$$\beta_s = \min \left[ \frac{E(r_{s,\min})}{E_p(T_s)}, \frac{E_{sv}(w)}{E_p(T_s)} \right]. \quad (\text{A9})$$

Is either of these correction factors likely to be better correlated with soil moisture than the other? In both cases, the first term in the brackets would be equal to unity if the minimal stomatal resistance were negligible compared to the aerodynamic resistance. In general, that term will be slightly smaller than unity, with the largest difference occurring for tall vegetation. In any case, this non-water-stressed branch of the  $\beta$  functions differs little between (A8) and (A9), so the comparison of these two functions should focus instead on the soil moisture-dependent branch, or the second term in the brackets. As was shown in the linear analysis above, the quantity  $E_p(T_w)$  is determined by the state of the atmosphere and the radiative properties of the surface, whereas  $E_p(T_s)$  depends on  $E_p(T_w)$ ,  $w$ , and  $T_a$ . At least for fixed  $E_p(T_w)$ ,  $\beta_w$  is then more closely correlated with  $w$  than  $\beta_s$  is, since the latter relation will exhibit scatter due to variations of air temperature. However  $E_p(T_w)$  is not a constant, and its variability will introduce considerable scatter into either relation. Hence, the physical theory suggests that Budyko's (1956) formulation may be marginally preferable, but that neither fits this physical situation well. In fact, Budyko's  $\beta_w$  function has been observed to vary seasonally, partly due to seasonal variation of  $E_p(T_w)$  but also due to predictable seasonal changes in the state of the vegetation. When one allows parametrically for such seasonal variation of the  $\beta$  function, the additional dependence of  $\beta_s$  on air temperature could probably be easily accommodated. Thus, one concludes again that there is little difference between (4) and (6) from a physical point of view.

### 4. Bare soil

Consider now the case of a homogeneous bare soil surface. Various theoretical analyses of the problem of moisture transport in soil support the idea that the evaporation rate will be either the potential evaporation rate or a soil-controlled rate, whichever is smaller. The soil-controlled rate is determined by the physical properties of the soil and, to first order, by the total water content in some well-defined layer of soil adjacent to the surface (Milly 1989). If there is no control of the evaporation by the soil, then evaporation is at the potential rate, and either definition of that rate may be



employed since they then coincide. Thus, for a given soil,

$$E = \min[E_p, E_s(w)], \quad (\text{A10})$$

in which  $E_s(w)$  is the soil-controlled rate of evaporation. We then find

$$\beta_s = \min\left[1, \frac{E_s(w)}{E_p(T_s)}\right] \quad (\text{A11})$$

and

$$\beta_w = \min\left[1, \frac{E_s(w)}{E_p(T_w)}\right]. \quad (\text{A12})$$

In both cases, the  $\beta$  function will vary in time due to variations of potential evaporation. The situation is similar to that considered in the preceding paragraph, although the  $\beta_w$  function is rather more stable over time since the soil properties are relatively constant over time. As a result, the use of (4) might be slightly preferable to the use of (6).

## 5. Subgrid variability

How might subgrid spatial heterogeneity of the land surface act to alter the conclusions reached above? One case of heterogeneity, which is the opposite extreme of the homogeneous cases already considered, is the case where the grid-scale average soil moisture  $w$  is distributed such that a fraction  $\omega$  of the grid cell is at field capacity and the remainder is dry. The areal fraction would then be given by

$$\omega = w/w_0, \quad (\text{A13})$$

in which  $w_0$  is the field capacity of the soil. The areal average evaporation rate  $\langle E \rangle$  from such an area is the weighted average of the rates from the two subareas. Consider first the case of full vegetal cover. No evaporation occurs from the dry region, but the wet region supplies water to the atmosphere at a rate given by (A5) with  $r_s$  equal to  $r_{s,\min}$ ,

$$\langle E \rangle = \omega E(r_{s,\min}). \quad (\text{A14})$$

In order to determine the grid-scale  $\beta_s$  in this situation, we must form the ratio

$$\begin{aligned} \beta_s &= \frac{\langle E \rangle}{\langle E_p(T_s) \rangle} = \frac{\langle E \rangle}{\omega E_p(T_w) + (1 - \omega) E_p(T_d)} \\ &= \frac{\omega E(r_{s,\min})}{E_p(T_w)[1 + (1 - \omega)\zeta]} \end{aligned} \quad (\text{A15})$$

in which  $T_d$  is the temperature of the dry surface and  $\zeta$  is as defined in (18). For this same situation,  $\beta_w$  may be calculated as

$$\beta_w = \frac{\langle E \rangle}{\langle E_p(T_w) \rangle} = \frac{\omega E(r_{s,\min})}{E_p(T_w)}. \quad (\text{A16})$$

We may assume that the ratio  $E(r_{s,\min})/E_p(T_w)$  varies slowly with the season and does not depart far below unity. Then, using (A13),  $\beta_w$  is a simple linear function of  $w$ , whereas the dependence of  $\beta_s$  on  $w$  is determined by the value of  $\zeta$  and hence also depends strongly on air temperature. In this case, (4) is clearly superior to (6). If the minimal stomatal resistance is small relative to the aerodynamic resistance (i.e., for short vegetation), then (A16) corresponds exactly to Budyko's empirically derived, field-scale  $\beta$ , given in (5), with  $w_k$  set equal to the field capacity.

Consider now the case of extreme subgrid variability in a bare soil. In analogy to (A15) and (A16), we may derive

$$\beta_s = \frac{\omega}{[1 + (1 - \omega)\zeta]} \quad (\text{A17})$$

$$\beta_w = \omega. \quad (\text{A18})$$

This result is essentially the same as that for a vegetated surface, and it can be seen that (4) would be preferable to (6), since the function  $\beta_w$  depends only on soil moisture.

## 6. Summary

Based on the simple analyses presented here, it appears that there is no physical basis to suggest that either (4) or (6) is superior to the other as a representation of evaporation from small, homogeneous land areas in nature, assuming that  $\beta$  is to be expressed as a simple function of soil moisture. However, (4) is clearly superior, under this assumption, if there is strong subgrid spatial variability of soil moisture. In that case, it can be shown that  $\beta_w$  is equal to the fractional saturation of soil moisture, while  $\beta_s$  depends also on the ratio  $\zeta$ , which is determined by several additional factors.

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