On NEMO 4.0 vertical scale factors interpolation

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This note proposes an update of *dom_vvl_interpol* routine with the addition of two vertical scale factors horizontal interpolation methods (integer parameter nmet=0,1,2 in the subroutine local definitions). These should work for any kind of vertical coordinates, including possibly ALE variants. For the seek of clarity, surface weighted interpolation is omitted below and only interpolation at U-points is considered.

1 Interpolation methods in the general case:

1.1 Default: (*nmet=0*)

That's the existing interpolation method in NEMO 4.0 which considers interpolation of scale factors anomalies:

$$e3u_{i+0.5,k} = e3u\_0_{i+0.5,k} + \frac{1}{2}\Big\{(e3t - e3t\_0)_{i,k} + (e3t - e3t\_0)_{i+1,k}\Big\} \tag{1}$$

As shown latter in the z* case, and in contrary to other methods below, this expression does no ensure the vertical sum equals the total depth at velocity points, i.e.:

$$\sum_{l=1}^{l=kbot} e3u_{i+0.5,l} \neq hu\_0_{i+0.5} + \frac{1}{2}(\eta_i + \eta_{i+1}) \tag{2}$$

Moreover, it does not deal with possibly negative thicknesses above steps, a situation which is likely to occur with ALE vertical coordinates. However, this method does work with s* coordinates, i.e. without steps. In that particular case, it is completely equivalent to the method 1 described below.

1.2 Method 1: Standard interface interpolation (*nmet=1*)

This is the most intuitive and straightforward way to get vertical scale factors at velocity points.

1. Compute interfaces depths (at W points):

$$
\begin{aligned}
z_{i,1} &= 0 \\
z_{i,k+1} &= \sum_{l=1}^{l=k} e3t_{i,l}
\end{aligned}
\tag{3}
$$

z equals 0 at the surface and $ht\_0 + \eta$ at the bottom.

2. Since interfaces are referenced to the local sea level, we shift them to the absolute (i.e. referenced to 0) vertical system:

$$
z'_{i,k} = z_{i,k} - \eta_i \tag{4}
$$

3. Then interpolate interfaces linearly at velocity points, with a specific condition at the bottom to ensure that the vertical integration of scale factors gives the correct depth at velocity points:

$$
\begin{aligned}
z'_{i+0.5,kbot} &= hu\_0_{i+0.5} \\
z'_{i+0.5,k} &= \frac{1}{2}\left\{ z'_{i,k} + z'_{i+1,k} \right\}
\end{aligned}
\tag{5}
$$

4. Lastly, we get the vertical scale factors:

$$
e3u_{i+0.5,k} = \frac{dz'_{i+0.5,k}}{dk} = z'_{i+0.5,k+1} - z'_{i+0.5,k} \tag{6}
$$

1.3 Method 2: Split external and internal interfaces (*nmet=2*)

Considering interfaces z issued from step 1 above, this time, we linearly rescale them according to the total depth $ht\_0 + \eta$:

$$
z''_{i,k} = z_{i,k} \frac{ht\_0_i}{ht\_0_i + \eta_i} \tag{7}
$$

Computations done in step 3 and 4 above are then performed identically. From this new set of scale factors, sea level anomaly contribution is then added in a similar fashion as it is done at T points in the z* coordinate case. One use sea level interpolated at velocity points to do so. This translates into:

$$
e3u_{i+0.5,k} = \frac{dz''_{i+0.5,k}}{dk} \left\{ 1 + \frac{1}{2} \frac{(\eta_i + \eta_{i+1})}{hu\_0_{i+0.5}} \right\} \tag{8}
$$

2 Interface crossings

To deal with interfaces crossing the bathymetry in methods 1 and 2 (this essentially occurs with ALE coordinates), The bottom interface is corrected as proposed by [Higdon, 2002] by taking a fraction of the T-point scale factor in the shallowest direction. This is illustrated in figure 1 with the use of the δ parameter that determines the minimum fraction used (0.8 in our case). In any case, when scanning the column from the bottom to the surface in computing interface heights, a minimum thickness of $10^{-10}m$ is assumed.

3 z* coordinate special case

Let's assume in the z* case that T-points thicknesses scale linearly with sea level anomaly:

$$e3t^*_{i,k} = e3t\_0_{i,k} \left\{ 1 + \frac{\eta_i}{ht\_0_i} \right\} \tag{9}$$

3.1 Default

Expanding formula 1, we get:

$$e3u^*_{i+0.5,k} = e3u\_0_{i+0.5,k} \left\{ 1 + \frac{1}{2} \frac{(\eta_i ht\_0_{i+1} + \eta_{i+1} ht\_0_i)}{ht\_0_i ht\_0_{i+1}} \right\} \tag{10}$$

Summing up the scale factors, it is obvious to see that the total depth is incorrect, unless the bathymetry is flat.

3.2 Method 1

Let's expand the vertical scale factors with z* coordinates with method 1. After some manipulation of the equations above, we first get the bottom vertical scale factor which is the only special case:

$$\begin{aligned} e3u^*_{i+0.5,kbot} = e3u\_0_{i+0.5,kbot} & \left\{ 1 + \frac{1}{2} \frac{(\eta_i ht\_0_{i+1} + \eta_{i+1} ht\_0_i)}{ht\_0_i ht\_0_{i+1}} \right\} \\ & + \frac{1}{2}\eta_i \left\{ 1 - \frac{hu\_0_{i+0.5}}{ht\_0_i} \right\} + \frac{1}{2}\eta_{i+1} \left\{ 1 - \frac{hu\_0_{i+0.5}}{ht\_0_{i+1}} \right\} \end{aligned} \tag{11}$$

With steps at the bottom only one of the last two terms in the right hand side is non zero (indeed $hu\_0_{i+0.5} = MIN(ht\_0_i, ht\_0_{i+1})$). Clearly the proportionality with the vertical scale factor at rest is lost. I guess this formulation can produce negative thicknesses with non negative total heights, right ?
For other levels, since we consider z coordinates, with assume $e3u\_0_{i+0.5,k} = e3t\_0_{i,k} = e3t\_0_{i+1,k}$ and obtain:

$$e3u^*_{i+0.5,k} = e3u\_0_{i+0.5,k} \left\{ 1 + \frac{1}{2} \frac{(\eta_i ht\_0_{i+1} + \eta_{i+1} ht\_0_i)}{ht\_0_i ht\_0_{i+1}} \right\} \tag{12}$$

which is identical to the default case (equation 10). Beside that, by rewritting equation 11 in this form:

$$\begin{aligned} e3u^*_{i+0.5,kbot} = e3u\_0_{i+0.5,kbot} & \left\{ 1 + \frac{1}{2} \frac{(\eta_i ht\_0_{i+1} + \eta_{i+1} ht\_0_i)}{ht\_0_i ht\_0_{i+1}} \right\} \\ & + \frac{1}{2}(\eta_i + \eta_{i+1}) - hu\_0_{i+0.5} \left\{ \frac{1}{2} \frac{(\eta_i ht\_0_{i+1} + \eta_{i+1} ht\_0_i)}{ht\_0_i ht\_0_{i+1}} \right\} \end{aligned} \tag{13}$$

one can easily check that the sum of vertical levels gives the expected total depth and the missing correction in the default interpolation method readily appears.
All in all, method 1 is identical to the default except at the bottom where an additional correction ensures that the total depth is correct.

3.3 Method 2

In the z* case, the vertical derivative in the right hand side of (8) is time invariant and equals the vertical scale factors at rest. Hence this formulation reverts to the same formula as for T-points, i.e.:

$$e3u^*_{i+0.5,k} = e3u\_0_{i+0.5,k}\left\{1 + \frac{1}{2}\frac{(\eta_i + \eta_{i+1})}{hu\_0_{i+0.5}}\right\} \tag{14}$$

Hence, as long as the total depth is positive, the vertical scale factor remains positive. One can also skip the computation of interfaces in that case and save some CPU and mpp communications.

References

[Higdon, 2002] R. L. Higdon, A two-level time stepping Method for layered ocean circulation models. *Journal of Computational Physics, 177:59-44 (2002).*

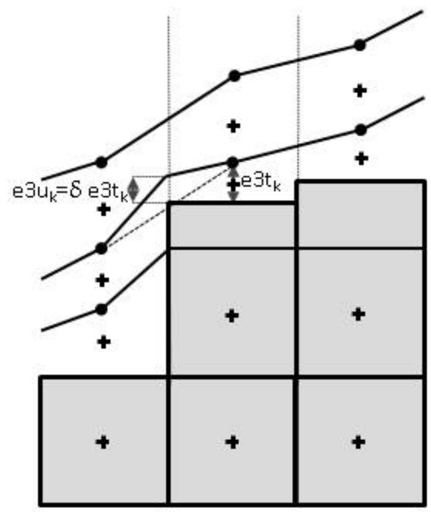

Figure 1: Trick from [Higdon, 2002] to ensure non zero bottom thicknesses at U-V points in case of an interface crossing a step. T-points: crosses, W-points: circles. Masked cells are in gray.