

The equation of deriving soil heat capacity in current model is:

The heat capacity of a soil is calculated as the sum of the heat capacities of its different constituents,

$$m_t C_t = m_d C_d + m_w C_w \quad (1)$$

The specific soil heat capacity is converted to volumetric heat capacity by dividing Eq. 30 with the total soil volume V_T ($V_T = V_d + V_w$),

$$\rho_t C_t = \rho_d C_d + \frac{m_w}{V_T} C_w = \rho_d C_d + \frac{V_w}{V_T} \rho_w C_w \quad (2)$$

ρ_t, ρ_d, ρ_w : the total bulk density, soil bulk density, water density (kg m^{-3}); C_t, C_d, C_w : the specific heat capacity for total, dry soil, water ($\text{Jkg}^{-1}\text{K}^{-1}$); $\rho_t C_t, \rho_d C_d, \rho_w C_w$: the volumetric heat capacity for total, dry soil, and water ($\text{Jm}^{-3}\text{K}^{-1}$); m_t, m_d, m_w : the masses in kg of total, dry soil and water, respectively (kg); V_T, V_d, V_w : the volume of total soil, dry soil and water (m^3).

Recalling the V_w/V_T term in Eq. 2 is the volumetric soil moisture θ , the equation for volumetric soil heat capacity is written by:

$$C_{v,t} = C_{v,d} + \theta C_{v,w} \quad (3)$$

$C_{v,t}, C_{v,d}, C_{v,w}$: the volumetric heat capacity for total, dry soil, and water ($\text{Jm}^{-3}\text{K}^{-1}$). The value for $C_{v,d}$ was obtained from *Pielke* [2002, 2013]. It varies with different soil types. $C_{v,w}$ is constant.

The problem is in equations (2) and (3). The ρ_d used by *Pielke* [2002, 2013] is not bulk density as defined by m_d/V_T , but it is defined by m_d/V_d . The corrected equations are:

$$\rho_t C_t = \frac{m_d}{V_T} C_d + \frac{m_w}{V_T} C_w = \frac{m_d}{V_d} \frac{V_d}{V_T} C_d + \frac{V_w}{V_T} \rho_w C_w = \rho_d \left(1 - \frac{V_w}{V_T}\right) C_d + \frac{V_w}{V_T} \rho_w C_w \quad (2')$$

$$C_{v,t} = (1 - \theta_s) C_{v,d} + \theta C_{v,w} \quad (3')$$

It means the $(1 - \theta)$ is missing in front of $C_{v,d}$.