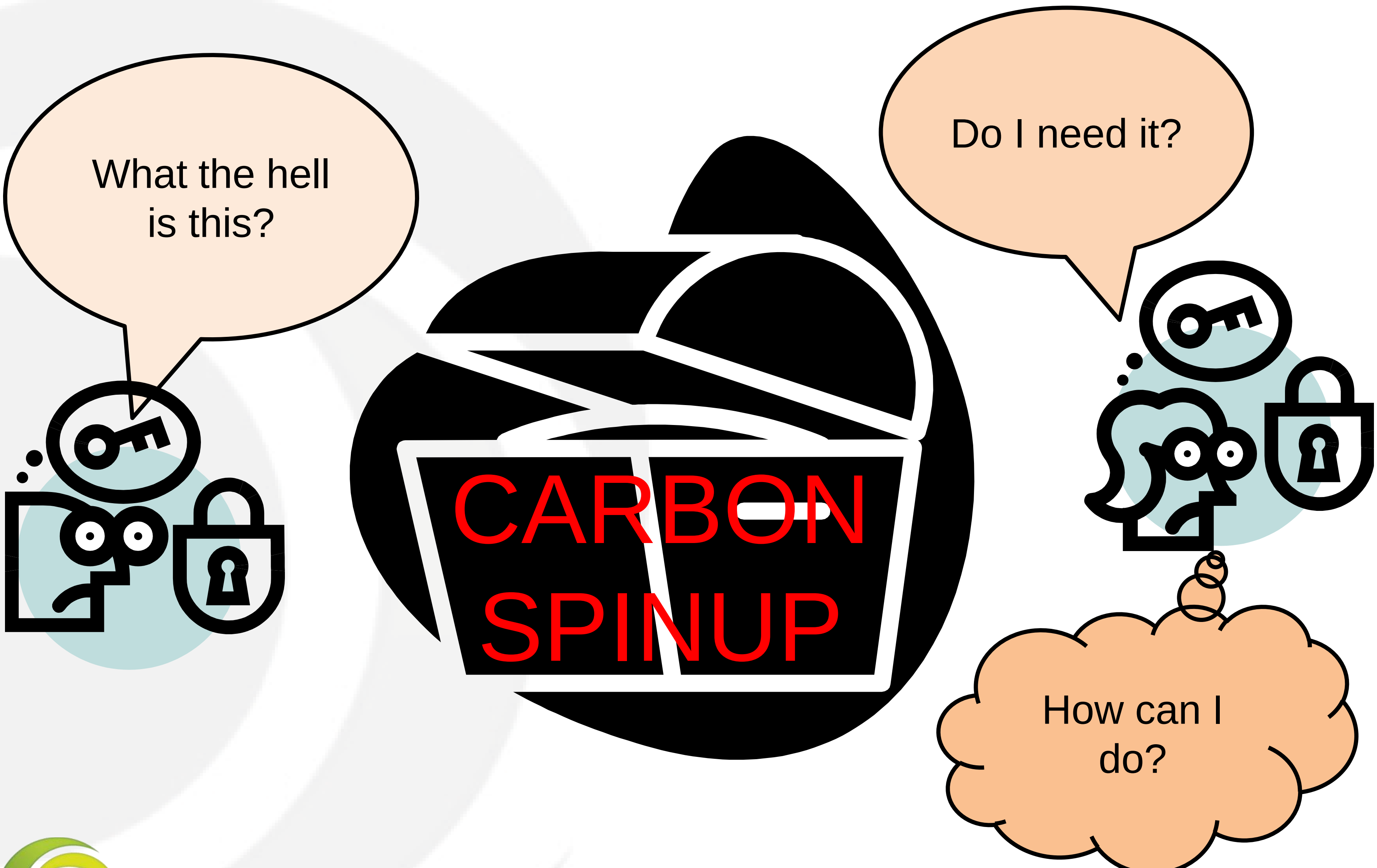
Introduction to the carbon spinup – Analytical solution



Carbon Spinup – Analytical solution

Scientific issue

Mathematical solution

Code implementation

libIGCM configuration

Implementation done by D. Solyga, N. Vuichard and J. Ghattas

Scientific issue

- Use of Terrestrial Biosphere Models?
- Steady state
- Carbon Spinup
- Dynamics of carbon pools
- Case with two carbon pools
- In ORCHIDEE
- CENTURY model

What is the use of Terrestrial Biosphere Models?

→ Test our understanding of processes.

→ Lead numerical experiments, for example introduction of disturbances such as:
- Climate Change
- Increasing atmospheric CO_2 concentration
- Land Use Change
- …

➲ A stable initialization state (with no trend) is required to study impacts.

Steady state

- Without disturbances, the carbon cycle in terrestrial ecosystems and in TBMs reaches an equilibrium, with all variables at a steady state (mean stability over a forcing period).
- This steady state equilibrium is usually used as the initialization state of experiments with TBM (especially in MIP).
- This ideal state is useful when you don't know the real history (plantation date, fires, LUC, …) so always with regional/global simulations.
- This equilibrium depends on climate, vegetation type, soil.

The carbon cycle is conceptually represented using carbon pools exchanging carbon.

The equilibrium is usually reached by running the carbon model **several thousands of years** to bring all carbon pools at equilibrium.

This operation is named as the **spinup** of the model.

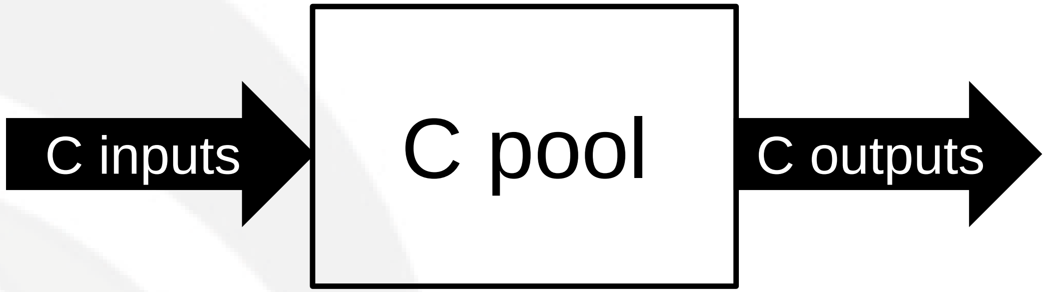
The computational cost is quite heavy as compared to the experiment itself (hundreds of years).

➲ We need to optimise the spinup.

➲ Analytical expression of the carbon stocks at the equilibrium state

Dynamics of carbon pools

Carbon processes in terrestrial ecosystems can be represented by **linear first-order differential equations**.



b influx

$$\frac{dC}{dt} = -aC + b$$

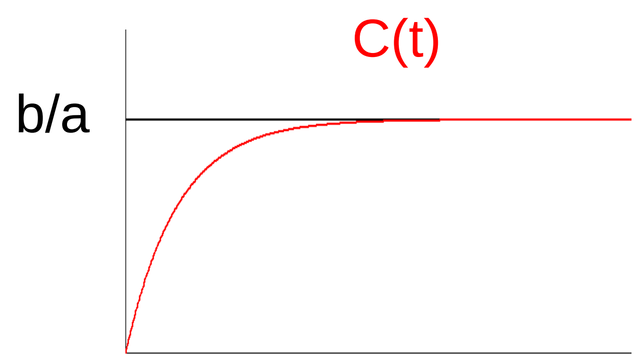
a decomposition rate (a>0)

$a = A\rho_T\rho_W$

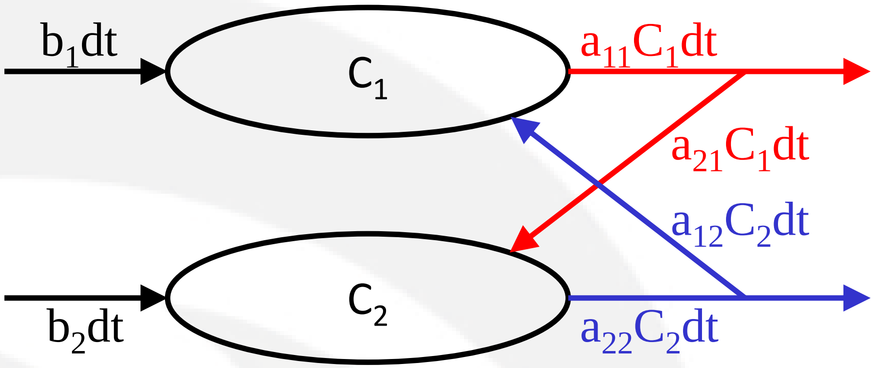
$\tau = 1/a$ residence time

Simple case: one pool, a and b constants, C(0)=0

➲ $$C(t) = \frac{b}{a}(1 - e^{-at})$$



Case with two carbon pools



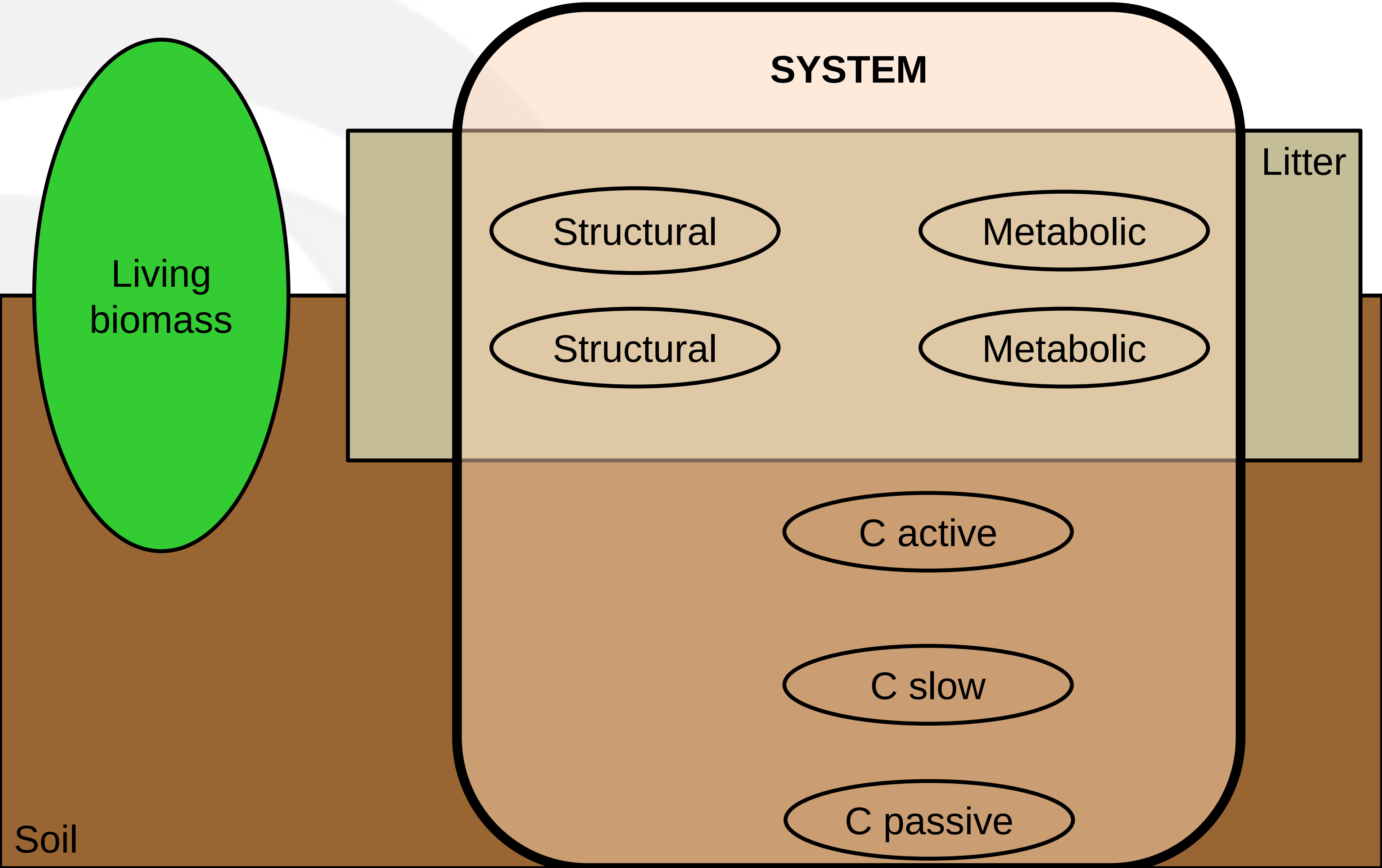
$$\frac{dC_1}{dt} = -a_{11}C_1 + a_{12}C_2 + b_1$$

$$\frac{dC_2}{dt} = a_{21}C_1 + a_{22}C_2 + b_2$$

$$\frac{d}{dt}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -a_{11} & a_{12} \\ a_{21} & -a_{22} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\frac{dC}{dt} = AC + B$$

In ORCHIDEE


SYSTEM
Litter
Structural
Metabolic
Structural
Metabolic
Living biomass
C active
C slow
C passive
Soil

CENTURY model

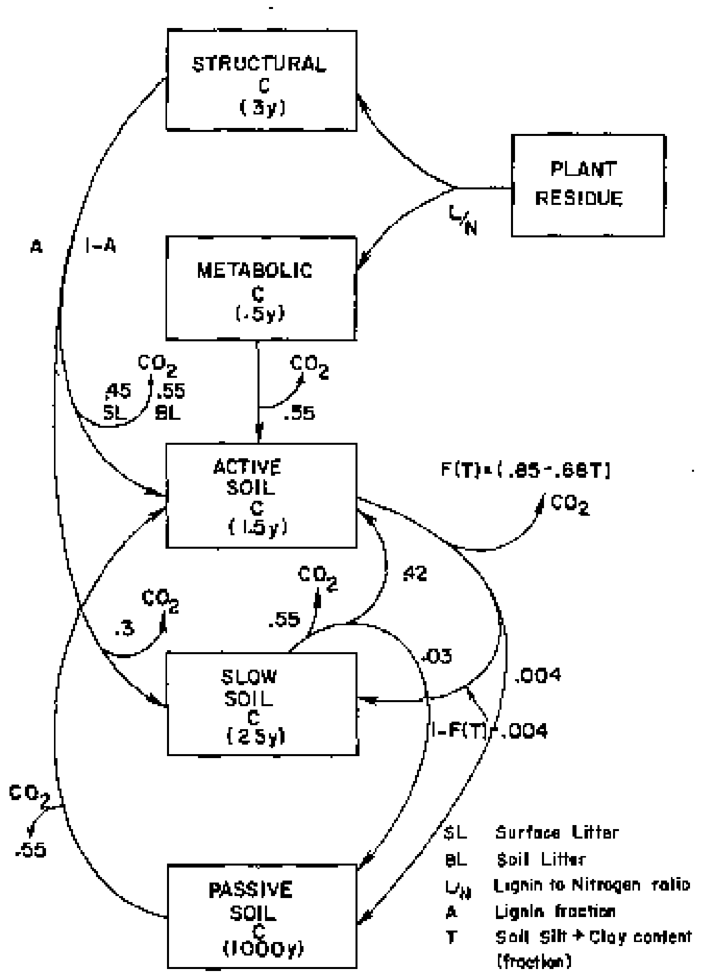


Fig. 1. Flow diagram for the C flows in the Century model.

Parton et al. (1987)

Mathematical implementation

- System dynamics
- Mathematical manipulations
- Steady-state analytical expression

Lardy et al. (2011)
C is the vector of carbon stocks.
The system dynamics is represented as:

$$C'(t) = \rho_t A_t C_t + B_t \quad (1)$$

with: $$C'(t) = \frac{C_{t+1} - C_t}{dt} \quad (2)$$

ρ_t represents the temperature and water stresses.
A_t is the matrix of the maximum decomposition rates.
B_t is the vector of inputs.

We rearrange (1) and (2) in:

$$C_{t+1} = (I + \rho_t dt A_t) C_t + B_t dt \quad (3)$$

and define:

$$D_t = (I + \rho_t dt A_t) \quad (4)$$

Some mathematical manipulations

$$C_{t+1} = (I + \rho_t dt A_t) C_t + B_t dt \qquad (3)$$

$$D_t = (I + \rho_t dt A_t) \qquad (4)$$

Equation (3) may thus be rewritten:

$$C_{t+1} = D_t C_t + B_t dt \qquad (5)$$

By induction we have:

$$C_t = \sum_{i=t_0}^{t-1} \left(\prod_{j=i+1}^{t-1} D_j \right) B_i dt + \left(\prod_{i=t_0}^{t-1} D_i \right) C_{t_0} \qquad (6)$$

We introduce the following series:

$$\begin{cases} V_{t_0} = D_{t_0} \\ V_t = D_t V_{t-1} \end{cases} \qquad (7) \qquad \text{and} \qquad \begin{cases} U_{t_0} = B_{t_0} dt \\ U_t = D_t U_{t-1} + B_t dt \end{cases} \qquad (8)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^{t} D_i \qquad (9) \qquad U_t = \sum_{i=t_0}^{t} \left(\prod_{j=i+1}^{t} D_j \right) B_i dt \qquad (10)$$

Steady-state analytical expression

$$C_t = \sum_{i=t_0}^{t-1} (\prod_{j=i+1}^{t-1} D_j) B_i dt + (\prod_{i=t_0}^{t-1} D_i) C_{t_0} \quad (6)$$

$$V_t = D_t V_{t-1} = \prod_{i=t_0}^{t} D_i \quad (9)$$

$$U_t = \sum_{i=t_0}^{t} (\prod_{j=i+1}^{t} D_j) B_i dt \quad (10)$$

Thus:

$$C_t = U_{t-1} + V_{t-1} C_{t_0} \quad (11)$$

At equilibrium:

$$C_t = C_{t_0} = C^* \quad (12)$$

$$C^* = U_{t-1} + V_{t-1} C^* \quad (13)$$

$$(I - V_{t-1}) C^* = U_{t-1} \quad (14)$$

$$C^* = (I - V_{t-1})^{-1} U_{t-1} \quad (15)$$

Code implementation

Initialization part

Loop part

Final inversion

constantes_var.f90
global variable spinup_analytic initialized to FALSE

constantes.f90::activate_sub_models
SPINUP_ANALYTIC KEYWORD read

stomate.f90::stomate_init
allocation of variables

IF spinup_analytic

stomate.f90::stomate_initialize
KEYWORDs read: SPINUP_PERIOD, EPS_CARBON

stomate_io.f90::readstart
Specific variables are read from the restart file:
'Global_years', 'nbp_sum', 'nbp_flux', 'ok_equilibrium', 'MatrixV','VectorU','previous_stock', 'current_stock'

Loop part

At each sechiba time step

stomate_io.f90::write_restart

IF (lstep_last) specific variables are written in the restart file.

stomate_litter.f90::littercalc

filling of MatrixA and VectorB with fluxes related to litter pools

stomate_soilcarbon.f90::soilcarbon

filling of MatrixA with fluxes related to soil carbon pools

A = A + I (=D)

At each stomate time step

lpj_fire.f90::fire

update of the terms of MatrixA related to above ground litter pools using firefrac

stomate.f90::stomate_main

nbp is accumulated.

stomate.f90::stomate_main

$V_t = D_t V_{t-1}$ $U_t = D_t U_{t-1} + B_t dt$

stomate.f90::stomate_main

IF EndOf Year

Increment the years counter (global_years).

IF global_years is a multiple of SPINUP_PERIOD

- $C^* = (I - V_{t-1})^{-1} U_{t-1}$ is computed using the Gauss-Jordan method (gauss_jordan_method.f90::gauss_jordan_method).
- Compute the relative error over the passive carbon pool (sum over all PFTs for each pixel, gauss_jordan_method.f90::error_L1_passive).
- For pixels where relative error < EPS_CARBON ok_equilibrium=TRUE
- Update all pools to new values.
- **IF all pixels at equilibrium: END OF THE ANALYTICAL SPINUP**

ENDIF

ENDIF

libIGCM SPINUP_ANALYTIC configuration

Configuration files

Expected evolution of NBP & CARBON_PASSIVE

```
#==========================================================================
#D-- UserChoices -
DateBegin=1901-01-01
DateEnd=2240-12-31
#
# Forcing data between 1901 and 1910
CyclicBegin=1901
CyclicEnd=1910
#==========================================================================
#D-- Post -
#D- Do we rebuild parallel output, this flag determines
#D- frequency of rebuild submission
RebuildFrequency=NONE → 5Y (if IOIPSL)
…
#==========================================================================
#D-- SRF - SECHIBA
[SRF]
WriteFrequency="1Y"

#==========================================================================
#D-- SRF - STOMATE
[SBG]
WriteFrequency="1Y"
```

orchidee_ol.card

```
[BoundaryFiles]
List=        (${R_IN}/SRF/METEO/CRU-NCEP/v5.3.2/twodeg/cruncep_twodeg_${CyclicYear}.nc, forcing_file.nc)
```

sechiba.card

```
[UserChoices]
# VEGET_UPDATE=0Y no change in vegetation map. PFTmap should be set only in InitialStateFiles/List.
# VEGET_UPDATE=1Y change vegetation map every year. PFTmap should be added in BoundaryFiles/List and
removed from InitialStateFiles/List.
VEGET_UPDATE=0Y

[InitialStateFiles]
List=     (${R_IN}/SRF/routing.nc, .),     \
        (${R_IN}/SRF/soils_param.nc, .), \
        (${R_IN}/SRF/soils_param_usdatop.nc, .), \
        (${R_IN}/SRF/cartepente2d_15min.nc, .), \
        (${R_IN}/SRF/floodplains.nc, .), \
        (${R_IN}/SRF/reftemp.nc, .), \
        (${R_IN}/SRF/albedo/alb_bg_modisopt_2D.nc, alb_bg.nc) , \
        (${R_IN}/SRF/PFTmap_1850to2005_AR5_LUHa.rc2/PFTmap_IPCC_1860.nc, PFTmap.nc)
```

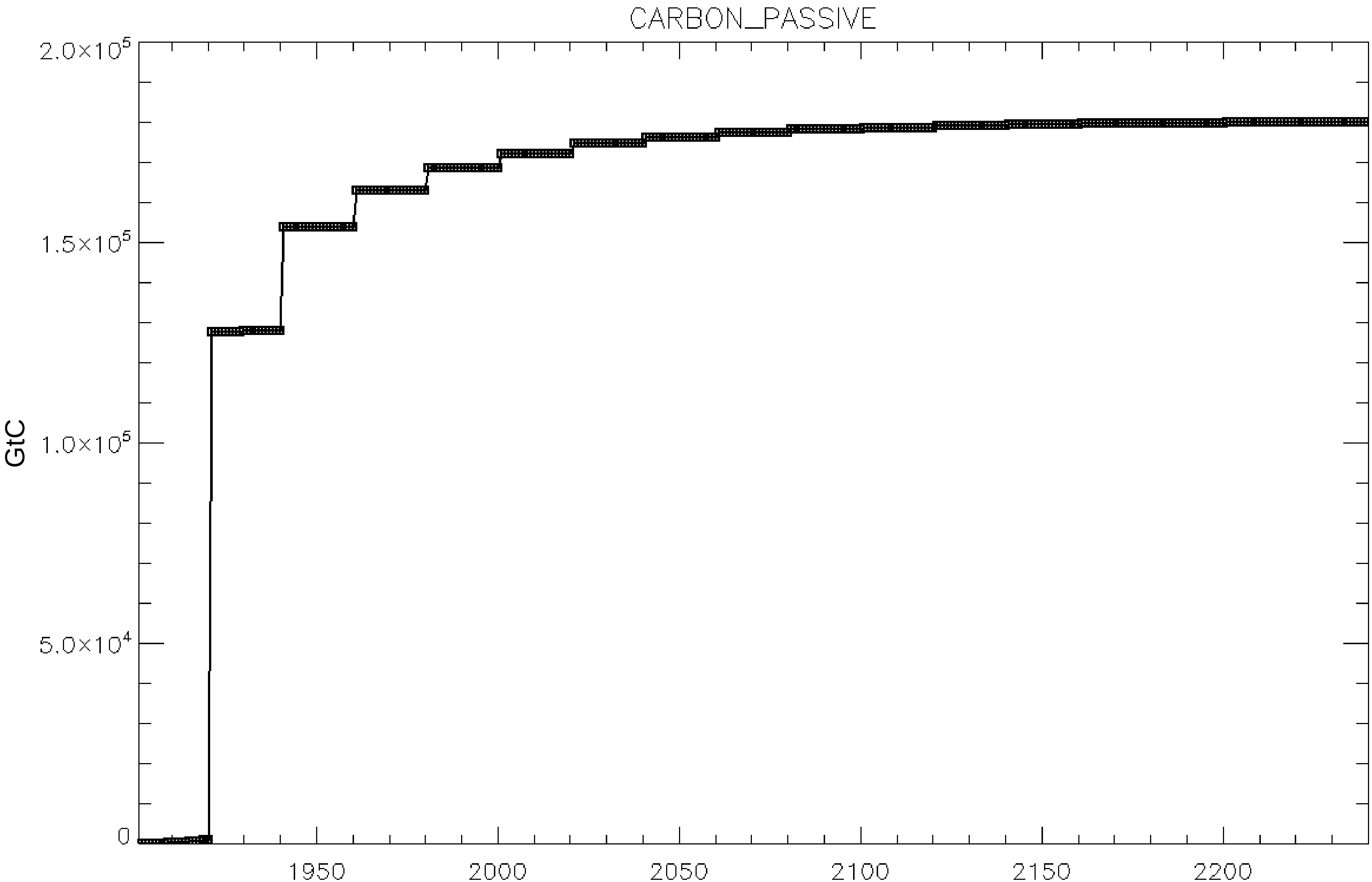
COMP (continued)

stomate.card

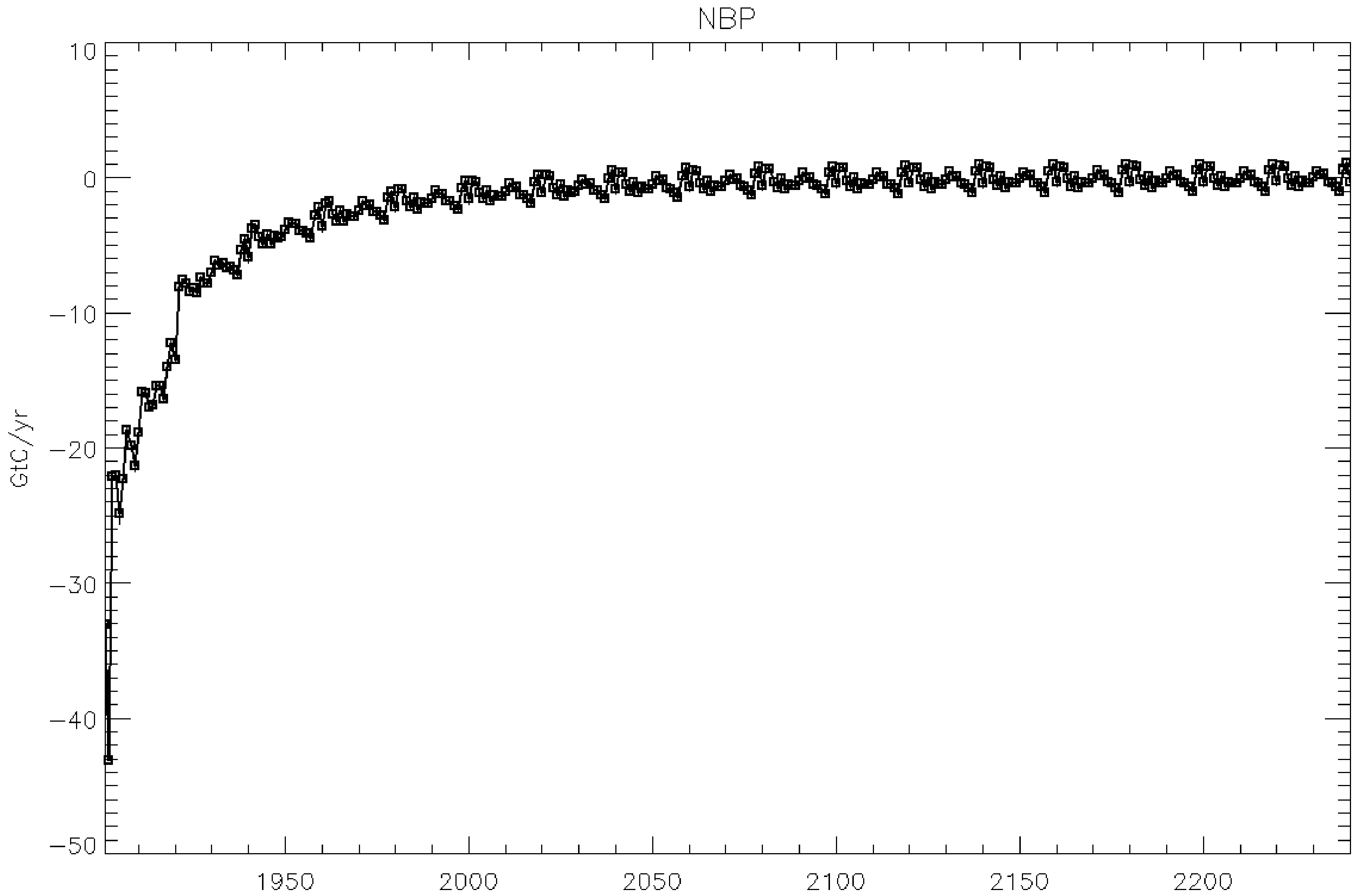
```
[UserChoices]
# SPINUP_ANALYTIC=y/n : Activate the spinup analytic option to solve the carbon in soil balance
SPINUP_ANALYTIC=y

[InitialStateFiles]
List=   ()

[BoundaryFiles]
List=          ()
ListNonDel= ()

[SmoothFiles]
List=          ()

[ParametersFiles]
List=   (${SUBMIT_DIR}/PARAM/run.def, .)

[RestartFiles]
# List restart that have to be saved/restored each loop (file out, saved, and in) :
List=          (stomate_rest_out.nc, stomate_rest.nc, stomate_rest_in.nc)
```

```
# SECHIBA history output level (0..10)
# default = 5
SECHIBA_HISTLEVEL = 11

# SECHIBA history output level (0..10)
# default = 5
SECHIBA_HISTLEVEL = 11

# Write frequency in days or -1 for monthly output in stomate_ipcc_history.nc
# default = 0.
STOMATE_IPCC_HIST_DT = _AUTO_

# ATM_CO2=287.14 : Year 1860 specified for TRENDY2 spinup
ATM_CO2 = _AUTO_: DEFAULT = 287.14

# Deactivate fire
FIRE_DISABLE=y

# EPS_CARBON ([%]   ) :  Allowed error on carbon stock  {SPINUP_ANALYTIC}
EPS_CARBON =  0.01

# SPINUP_PERIOD ([years]   ) :  Period to calculate equilibrium during spinup analytic {SPINUP_ANALYTIC}
SPINUP_PERIOD =  -1
```

CARBON_PASSIVE time-series global scale



NBP time-series global scale



Useful references / link

- Odum, E.P., 1969. The strategy of ecosystem development, Science, 164, 262-270.
- Parton, W.J., Schimel, D.S., Cole, C.V. and Ojima, D.S., 1987. Analysis of Factors Controlling Soil Organic-Matter Levels in Great-Plains Grasslands. Soil Science Society of America Journal, 51:1173-1179.
- Thornton, P.E. and Rosenbloom, N.A., 2005. Ecosystem model spin-up: Estimating steady state conditions in a coupled terrestrial carbon and nitrogen cycle model. Ecological Modelling, 189:25-48.
- Lardy, R., Bellocchi, G. and Soussana, J.F., 2011. A new method to determine soil organic carbon equilibrium. Environmental Modelling & Software, 26:1759-1763.
- Luo, Y.Q. and Weng, E.S., 2011. Dynamic disequilibrium of the terrestrial carbon cycle under global change. Trends in Ecology & Evolution, 26:96-104.
- Xia, J.Y., Luo, Y.Q., Wang, Y.P., Weng, E.S. and Hararuk, O., 2012. A semi-analytical solution to accelerate spin-up of a coupled carbon and nitrogen land model to steady state. Geoscientific Model Development, 5:1259-1271.

Work of D. Solyga:
http://forge.ipsl.jussieu.fr/orchidee/wiki/DevelopmentActivities/AccelerationSpinup